Course Notes

Mathematical Models, Fall 2019

Queens College, Math 245

Prof. Christopher Hanusa

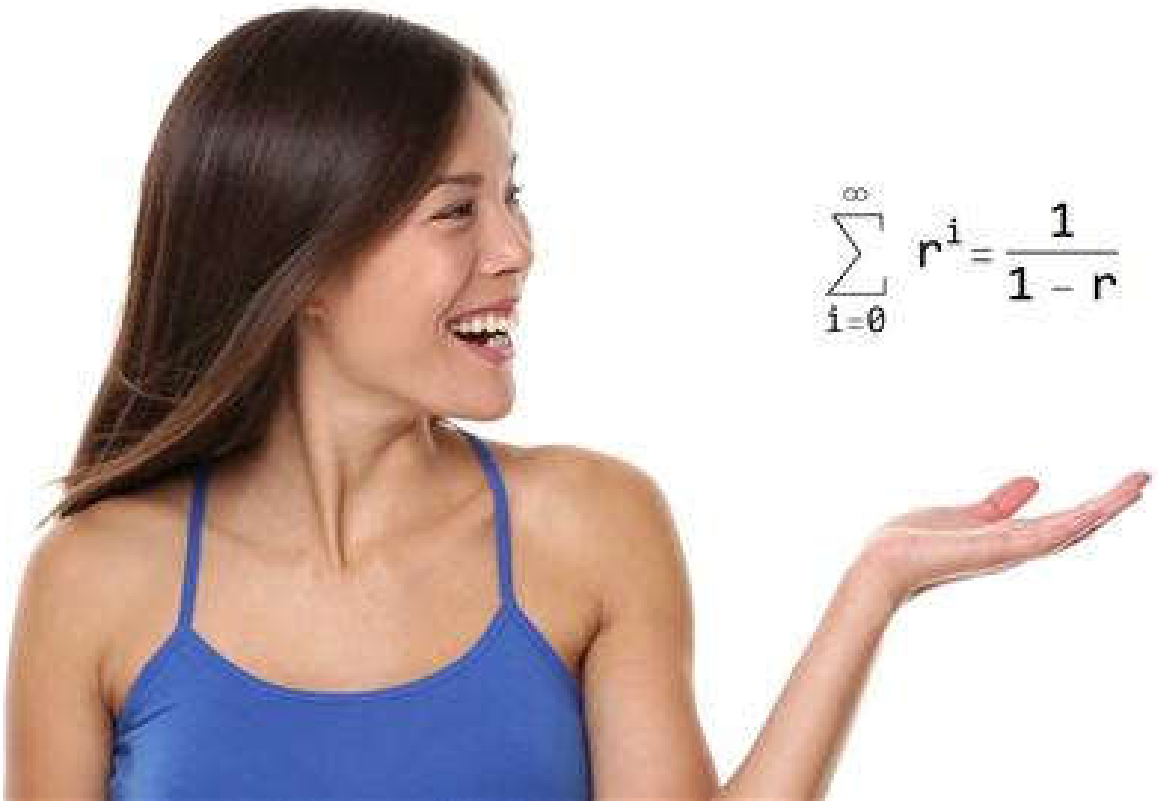
http://qcpages.qc.cuny.edu/~chanusa/courses/245/19/

What is mathematical modeling?

Is it....

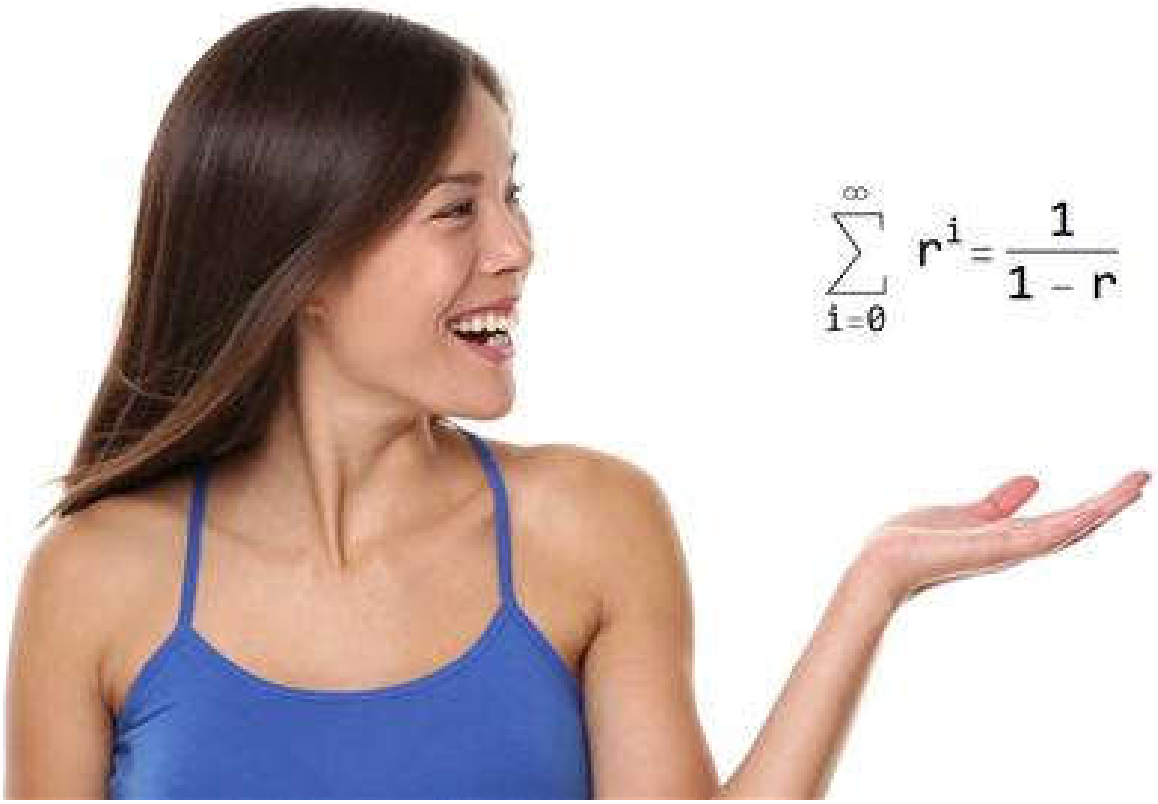
What is mathematical modeling?

Is it....



What is mathematical modeling?

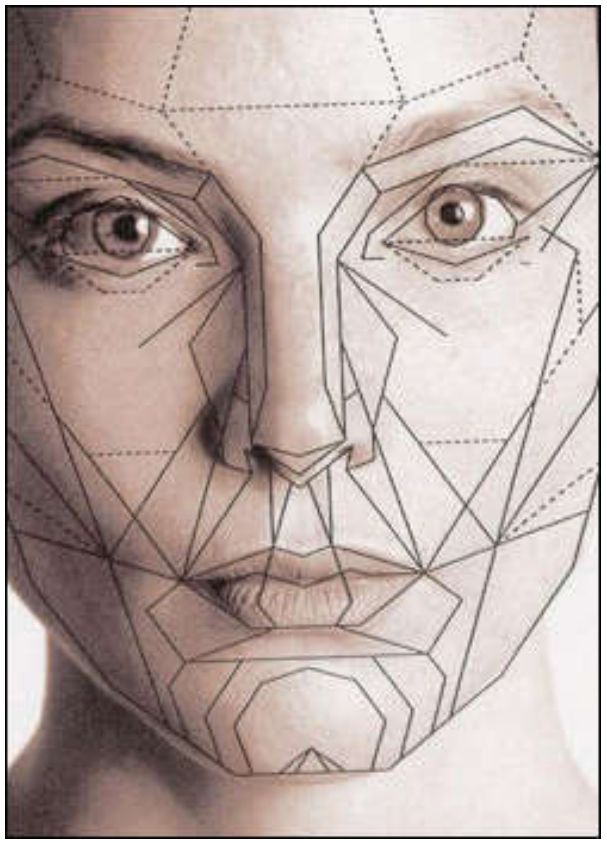
Is it....



No, that's modeling mathematics.

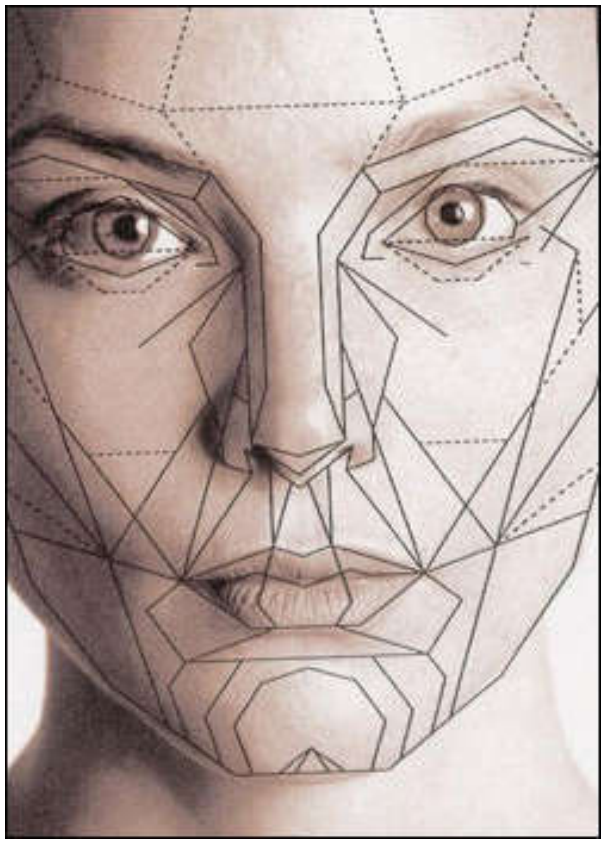
What is mathematical modeling?

Is it....



What is mathematical modeling?

Is it....



No, that's the mathematics of modeling.

What is mathematical modeling?

Is it....

What is mathematical modeling?

Is it....

No, that's modeling mathematical jewelry.

What is mathematical modeling?

Is it....

What is mathematical modeling?

Is it....

No, that's a model airplane.

What is mathematical modeling?

Is it....

No, that's a model airplane.

But we're getting closer.

A definition

What is a model?

A definition

What is a model?

- Miniature representation of something

A definition

What is a model?

- Miniature representation of something (Model airplane)

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation (Sewing pattern)

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation (Sewing pattern)
- An analogy to aid in visualization / conceptualization

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation (Sewing pattern)
- An analogy to aid in visualization / conceptualization (Drawing of an atom; Pendulum for public opinion.)

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation (Sewing pattern)
- An analogy to aid in visualization / conceptualization (Drawing of an atom; Pendulum for public opinion.)
- A detailed description of a situation.

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation (Sewing pattern)
- An analogy to aid in visualization / conceptualization
  (Drawing of an atom; Pendulum for public opinion.)
- A detailed description of a situation.
  (System of assumptions, data, inferences describing a situation)

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation (Sewing pattern)
- An analogy to aid in visualization / conceptualization
  (Drawing of an atom; Pendulum for public opinion.)
- A detailed description of a situation.
  (System of assumptions, data, inferences describing a situation)

A **mathematical model** is a representation of a real-world situation using mathematical terms.

A definition

What is a model?

- Miniature representation of something (Model airplane)
- An example for imitation or emulation (Sewing pattern)
- An analogy to aid in visualization / conceptualization
  (Drawing of an atom; Pendulum for public opinion.)
- A detailed description of a situation.
  (System of assumptions, data, inferences describing a situation)

A **mathematical model** is a representation of a real-world situation using mathematical terms.

We will see: **Math is Everywhere.**

Models underlie many real-world situations

- Countdown clocks: When will the next bus/train arrive?
- Product management: How many items will a store sell?
- Sea rise: How much higher will the oceans be in 50 years?
- Population: How many people will live in NYC in 2030?
- Student demand: How many sections of Math 245 to run?

A justification

Why do we model?

A justification

Why do we model?

$\rightarrow$ observed real-world

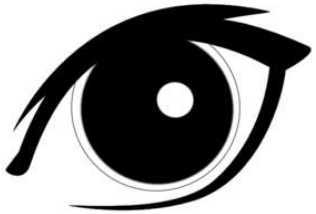
$\rightsquigarrow$ behavior

$\rightarrow$

As scientists, we want to understand how the world works.

A justification

Why do we model?



$\longrightarrow$ observed real-world

$\rightsquigarrow$ behavior

$\longrightarrow$

As scientists, we want to understand how the world works.

- What is happening?

A justification

Why do we model?

$\longrightarrow$ observed real-world

$\rightsquigarrow$ behavior

$\longrightarrow$

As scientists, we want to understand how the world works.

- What is happening?
- What are the reasons for the behavior?

A justification

Why do we model?

$\rightarrow$ observed real-world

$\rightsquigarrow$ behavior

$\rightarrow$

As scientists, we want to understand how the world works.

- What is happening?
- What are the reasons for the behavior?
- How do we convey that our reasoning is plausible?

A justification

Why do we model?

$\longrightarrow$ observed real-world

$\rightsquigarrow$ behavior

$\longrightarrow$

As scientists, we want to understand how the world works.

- What is happening?
- What are the reasons for the behavior?
- How do we convey that our reasoning is plausible?

Modeling is an **ACTIVE** process.

A justification

Why do we model?

observed real-world behavior

As scientists, we want to understand how the world works.

- What is happening?
- What are the reasons for the behavior?
- How do we convey that our reasoning is plausible?

Modeling is an **ACTIVE** process.

- Requires in-depth thought in order to understand and convey the essence of a situation.

A justification

Why do we model?

$\rightarrow$ observed real-world

$\rightsquigarrow$ behavior

$\rightarrow$

As scientists, we want to understand how the world works.

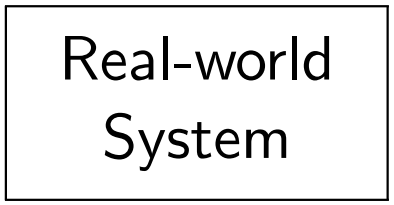
- What is happening?
- What are the reasons for the behavior?
- How do we convey that our reasoning is plausible?

Modeling is an **ACTIVE** process.

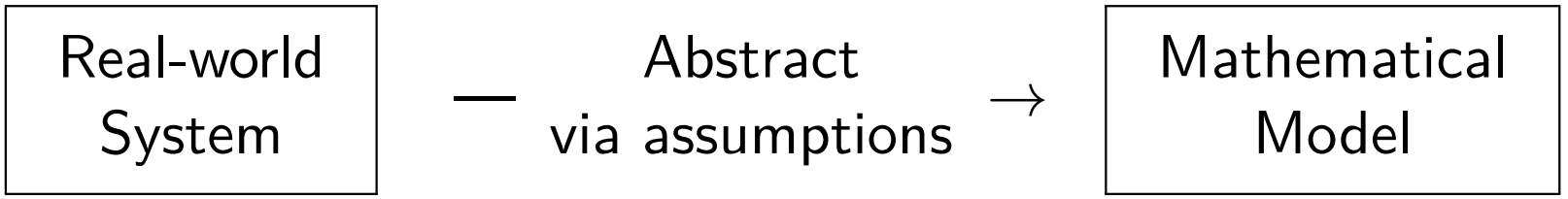
- Requires in-depth thought in order to understand and convey the essence of a situation.

In this class: We are going to create computer simulations in Python and use the language of mathematics to model the real world.

How do we model?

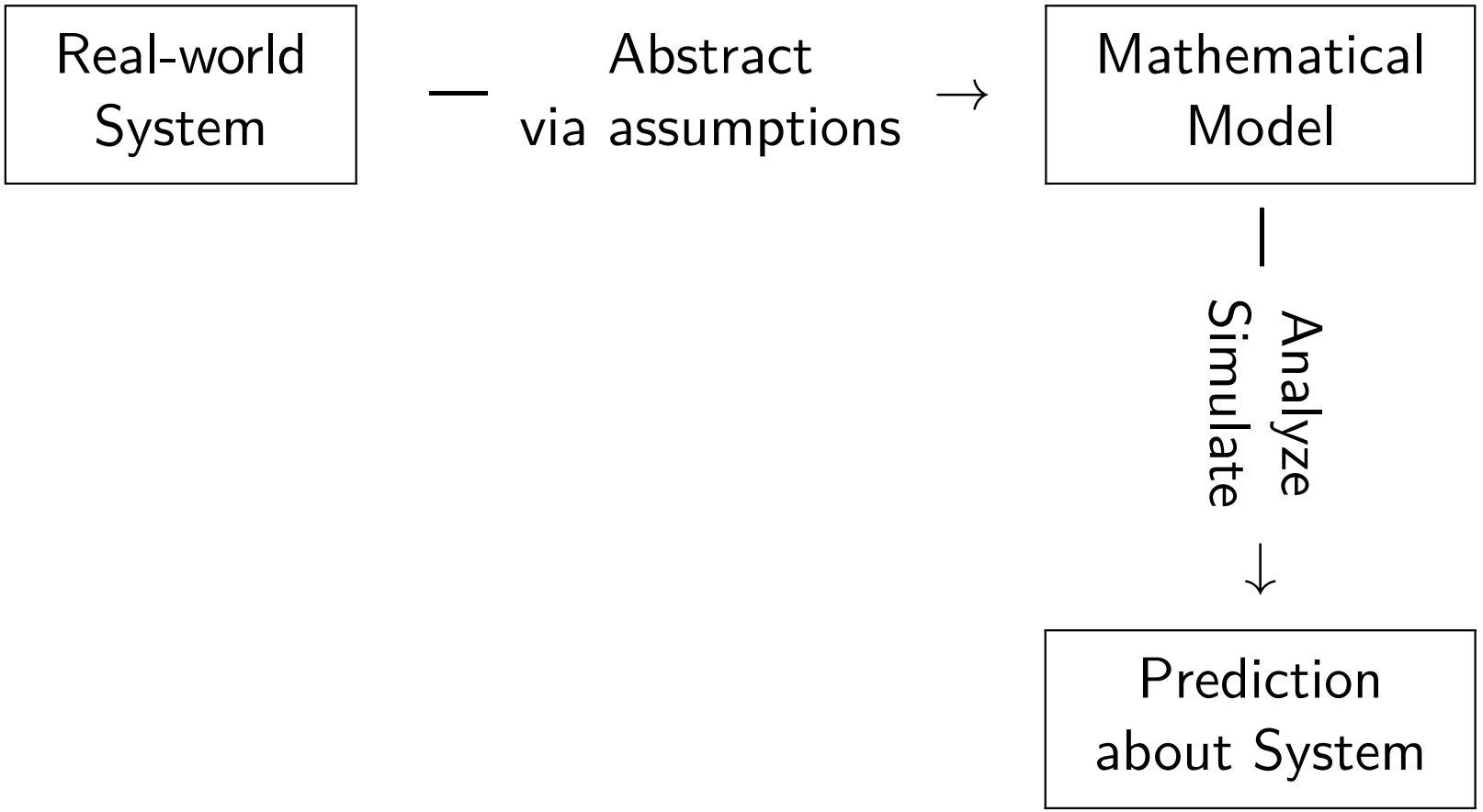


How do we model?



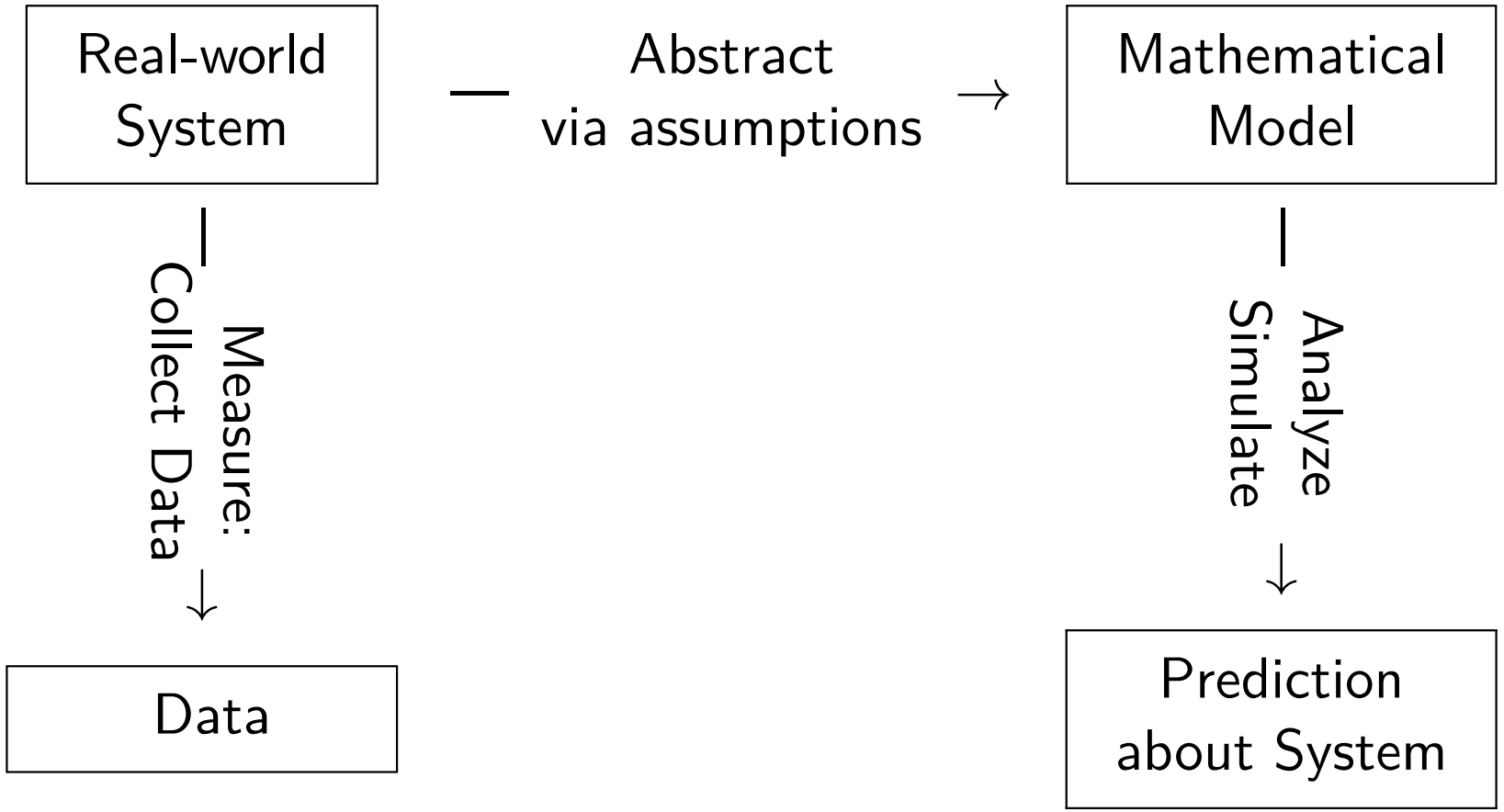
- Identify the most important variables in a real-world system

How do we model?



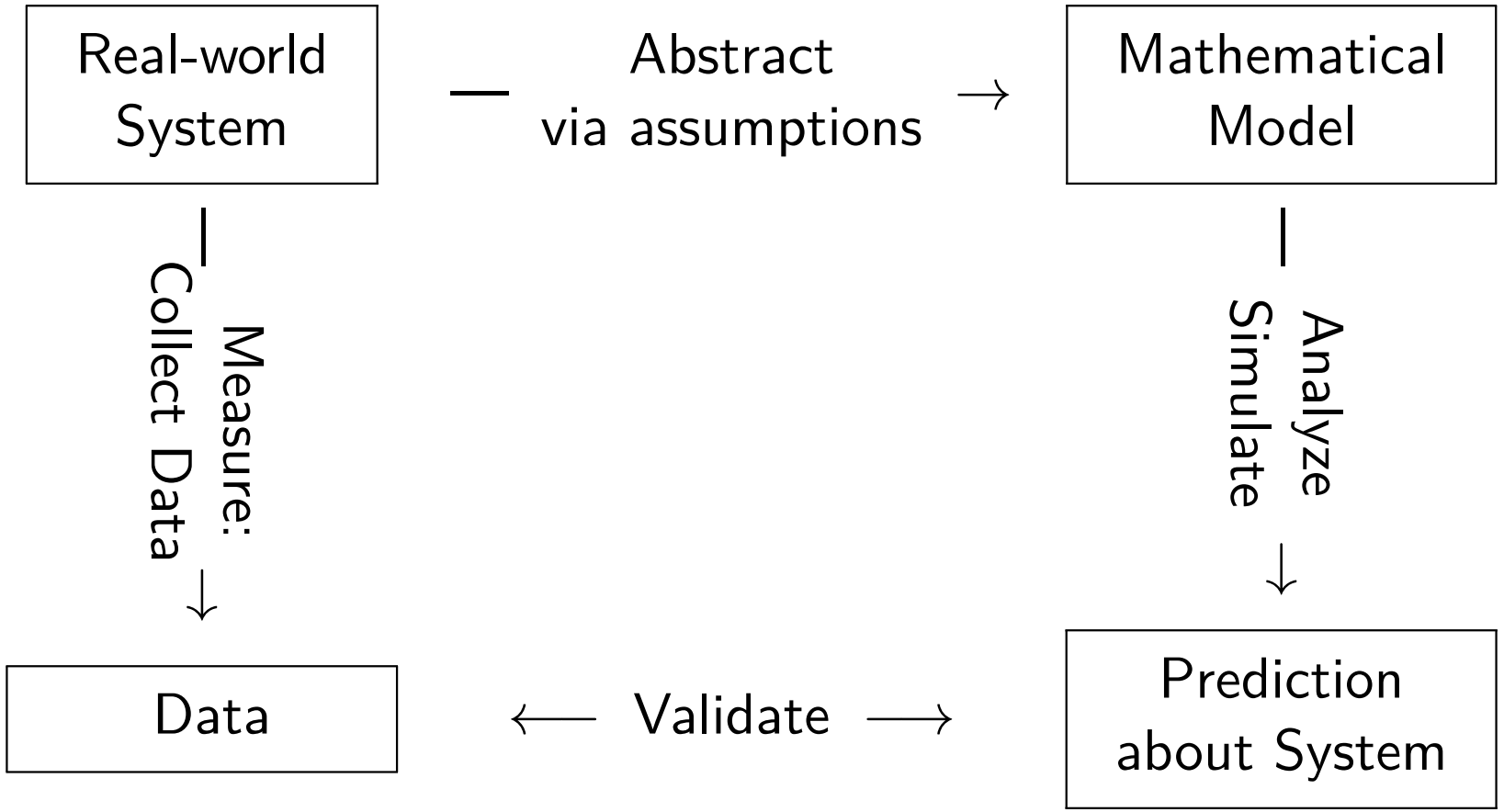
- Identify the most important variables in a real-world system
- Analyze the model / Create a computer simulation

How do we model?



- Identify the most important variables in a real-world system
- Analyze the model / Create a computer simulation
- Collect some data from the real world system

How do we model?



- Identify the most important variables in a real-world system
- Analyze the model / Create a computer simulation
- Collect some data from the real world system
- Validate your model and refine / revise!

A normal day in this class

- Arrive on time & Be ready to participate!

A normal day in this class

- Arrive on time & Be ready to participate!
- Discussion of homework questions, Daily Question.

A normal day in this class

- Arrive on time & Be ready to participate!
- Discussion of homework questions, Daily Question.
- New content
  - Theory and Practice of Modeling
  - Algorithm design and coding details
  - Take notes!

A normal day in this class

- Arrive on time & Be ready to participate!
- Discussion of homework questions, Daily Question.
- New content
  - Theory and Practice of Modeling
  - Algorithm design and coding details
  - Take notes!
- Project Work
  - Dedicated time to make progress and ask questions on project

A normal day in this class

In class

- Arrive on time & Be ready to participate!
- Discussion of homework questions, Daily Question.
- New content
  - Theory and Practice of Modeling
  - Algorithm design and coding details
  - Take notes!
- Project Work
  - Dedicated time to make progress and ask questions on project

A normal day in this class

Outside class

In class

- Arrive on time & Be ready to participate!
- Discussion of homework questions, Daily Question.
- New content
  - Theory and Practice of Modeling
  - Algorithm design and coding details
  - Take notes!
- Project Work
  - Dedicated time to make progress and ask questions on project

Outside class

A normal day in this class

Outside class

In class

- Arrive on time & Be ready to participate!
- Discussion of homework questions, Daily Question.
- New content
  - Theory and Practice of Modeling
  - Algorithm design and coding details
  - Take notes!
- Project Work
  - Dedicated time to make progress and ask questions on project

Outside class

- Learning after class
  - Review notes, Do homework, Project work

A normal day in this class

Outside class

- Preparing for class
  - Pre-reading, Answer Daily Question, Prepare questions.

In class

- Arrive on time & Be ready to participate!
- Discussion of homework questions, Daily Question.
- New content
  - Theory and Practice of Modeling
  - Algorithm design and coding details
  - Take notes!
- Project Work
  - Dedicated time to make progress and ask questions on project

Outside class

- Learning after class
  - Review notes, Do homework, Project work

Class Notebook

- Acquire a notebook (50–80 pages) and bring it to class everyday. (Quarantine it from other classes.)
- I will collect them every few weeks and check for completion.

Class Notebook

- Acquire a notebook (50–80 pages) and bring it to class everyday. (Quarantine it from other classes.)
- I will collect them every few weeks and check for completion.
- Label pages with the date and label each question.

Question 1-1. (problem statement here)

Answer the question in complete sentences.

(Leave some space for notes from discussion.)

Question 1-2. (problem statement here)

⋮

To do well in this class:

- **Form good study groups.**
  - Discuss homework and classwork.
  - Bounce around ideas, topics, questions.
  - It helps to have people to talk through things with.

To do well in this class:

- **Form good study groups.**
  - Discuss homework and classwork.
  - Bounce around ideas, topics, questions.
  - It helps to have people to talk through things with.
- **Put in the time OUTSIDE class.**
  - Three credits = 6–9 hours / week out of class.
  - Homework stresses key concepts from class; learning takes time.

To do well in this class:

- **Form good study groups.**
  - Discuss homework and classwork.
  - Bounce around ideas, topics, questions.
  - It helps to have people to talk through things with.
- **Put in the time OUTSIDE class.**
  - Three credits = 6–9 hours / week out of class.
  - Homework stresses key concepts from class; learning takes time.
- **Come to class prepared.**
  - Review previous day's notes.
  - Do the homework & work on your projects.

To do well in this class:

- **Form good study groups.**
  - Discuss homework and classwork.
  - Bounce around ideas, topics, questions.
  - It helps to have people to talk through things with.
- **Put in the time OUTSIDE class.**
  - Three credits = 6–9 hours / week out of class.
  - Homework stresses key concepts from class; learning takes time.
- **Come to class prepared.**
  - Review previous day's notes.
  - Do the homework & work on your projects.
- **Stay in contact.**
  - If you are confused, ask questions (in class and out).
  - Don't fall behind in coursework or homework.
  - I need to understand your concerns.

To do well in this class:

- **Form good study groups.**
  - Discuss homework and classwork.
  - Bounce around ideas, topics, questions.
  - It helps to have people to talk through things with.
- **Put in the time OUTSIDE class.**
  - Three credits = 6–9 hours / week out of class.
  - Homework stresses key concepts from class; learning takes time.
- **Come to class prepared.**
  - Review previous day's notes.
  - Do the homework & work on your projects.
- **Stay in contact.**
  - If you are confused, ask questions (in class and out).
  - Don't fall behind in coursework or homework.
  - I need to understand your concerns.

Everything posted online; first one (many parts) due Wednesday.

Meet the modelers

Group Activity. Get into groups of four-ish people, with people you don't know. Take some time to get to know them:

- Introduce yourself. (your name, where you're from, your major)
- Fill out **the blank side of** your notecard:
  - Write your name. (Stylize if you wish.)
  - Write a few words about your name to help me remember.
  - *Draw* something in the remaining space.
- Discuss with your groupmates why you wrote what you wrote.
- Exchange contact information. (phone / email / other)
- *Small talk suggestion:* What kept you busy this summer?

Meet the modelers

Group Activity. Get into groups of four-ish people, with people you don't know. Take some time to get to know them:

- Introduce yourself. (your name, where you're from, your major)
- Fill out **the blank side of** your notecard:
  - Write your name. (Stylize if you wish.)
  - Write a few words about your name to help me remember.
  - *Draw* something in the remaining space.
- Discuss with your groupmates why you wrote what you wrote.
- Exchange contact information. (phone / email / other)
- *Small talk suggestion:* What kept you busy this summer?

Thought Question. Brainstorm real-world situations where you interact with a mathematical model. Choose one. How does the mathematical model (or its conclusions) impact your life?

We are going to learn math modeling and python together.

Let's get Started!

- Grab a computer / Bring your own.

- Go to our course webpage:
  `qcpages.qc.cuny.edu/~chanusa/courses/245/19/`
- Find the day's plan $>$ `Content`.
- Head to $>$ `Software`.
- Use your Office365 account to access Azure Notebooks.
- Import the python notebooks from GitHub.
- While they are importing, access Google Classroom.
- Take a minute to answer the Daily Question.

Jupyter notebook advice

Jupyter is the notebook environm't. **Python** is the progr. language.

- **Always work in the `code` directory.**
- Make a new copy of the notebook before any modifications.

- Each time we start the server, all previous definitions are lost.
- Use the Python 3 kernel, not the Python 3.6 kernel.

- Jupyter notebooks look linear. They are not.
- Always evaluate the cells in order from top to bottom.

Let's flip a coin!

- We are using the `modsim` package; it must be imported each time we open the notebook.
- `modsim` relies on the `pint` package, so load it first.