Course Notes

Mathematical Models, Spring 2020

Queens College, Math 245

Prof. Christopher Hanusa

http://qcpages.qc.cuny.edu/~chanusa/courses/245/20/

What is mathematical modeling?

Is it....

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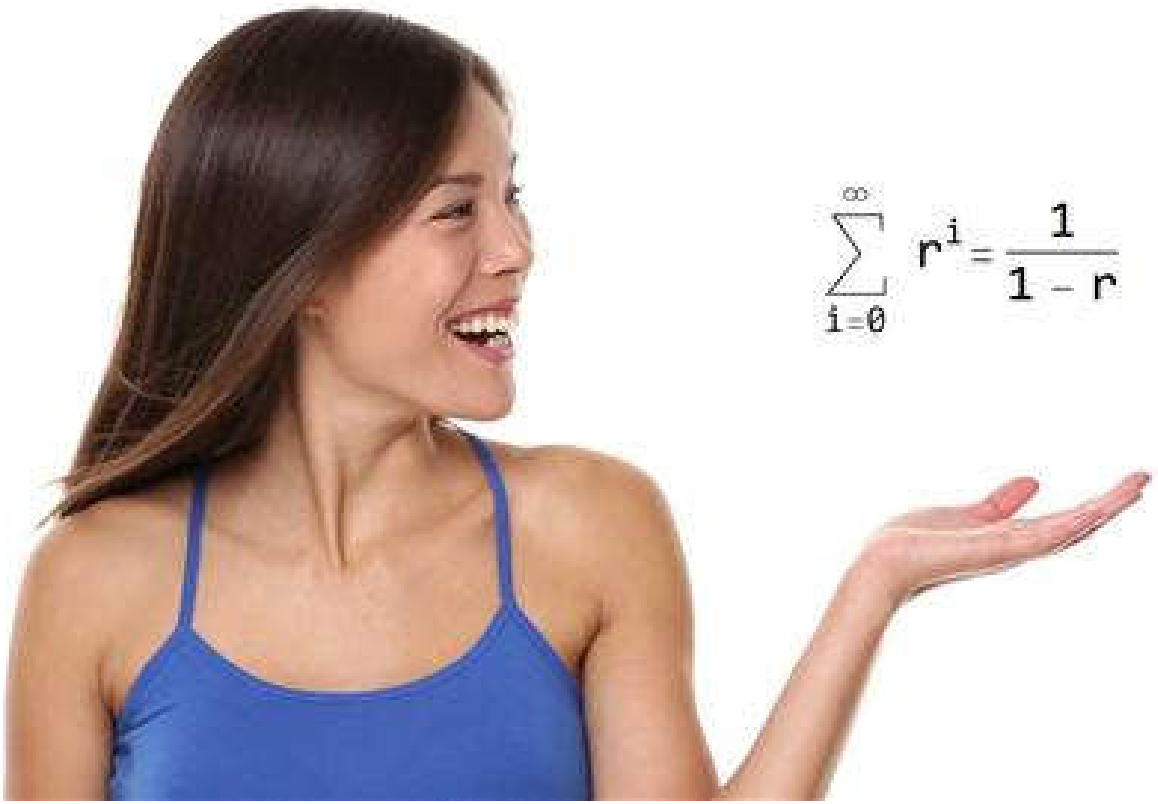

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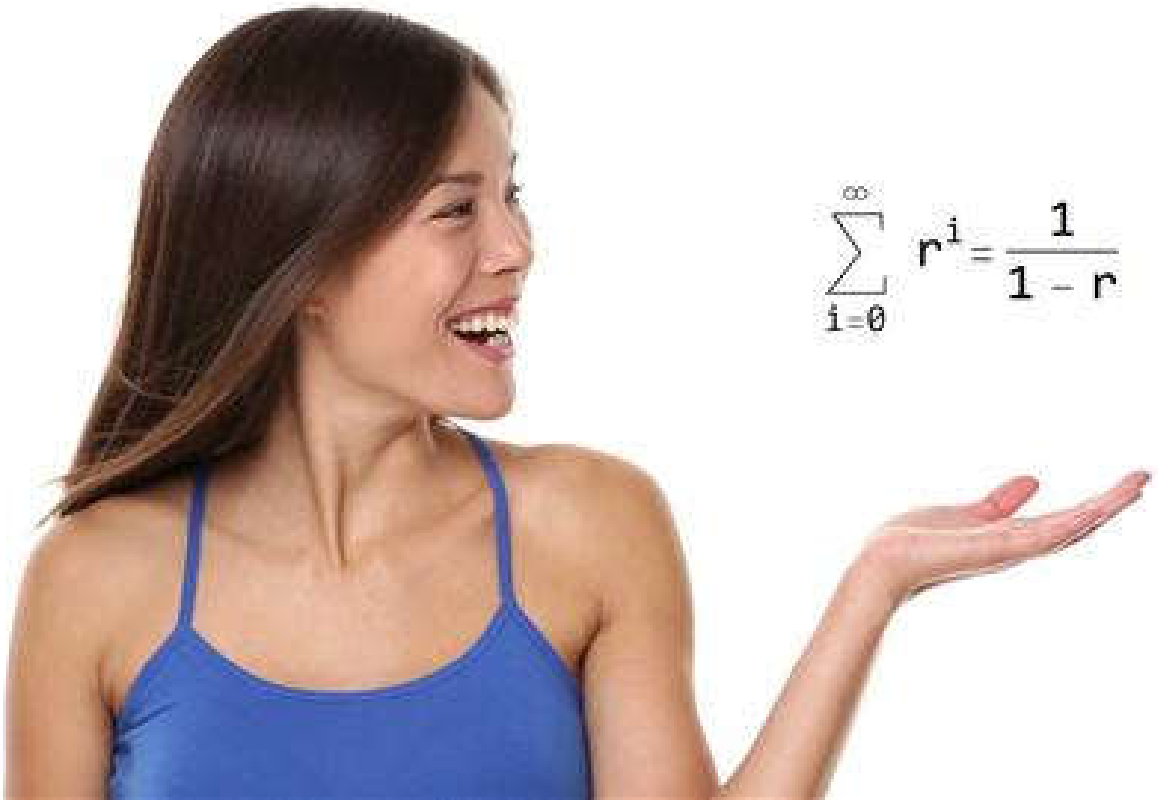

No, that's modeling mathematics.

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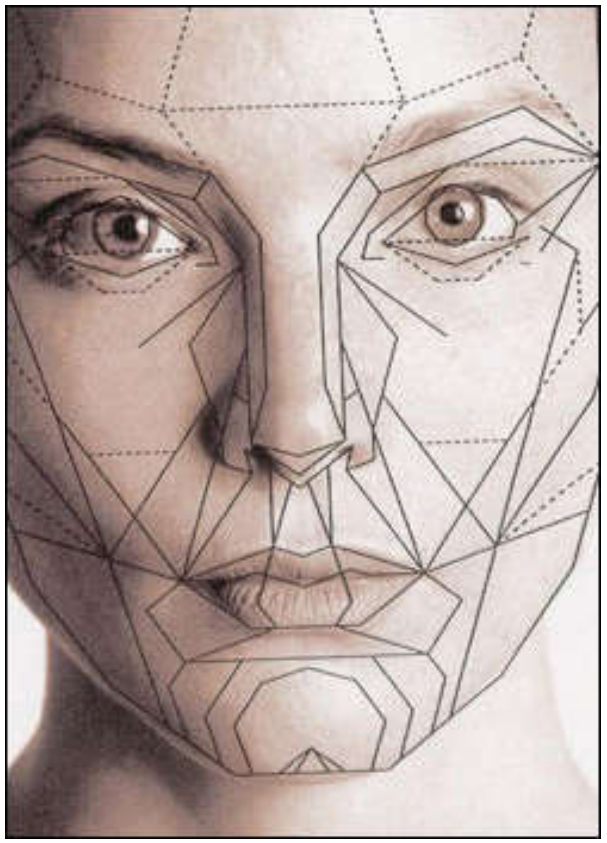

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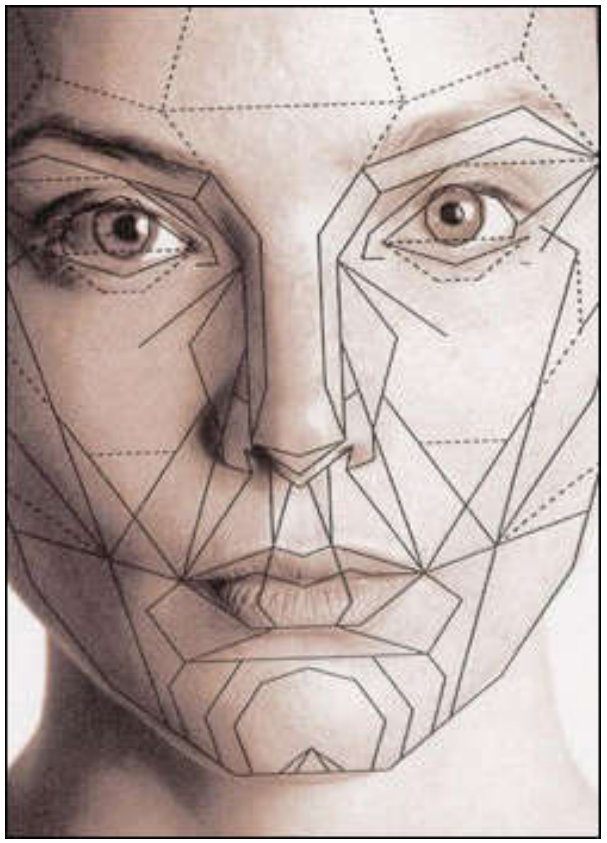

No, that's the mathematics of modeling.

What is mathematical modeling?

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No, that's modeling mathematical jewelry.

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No, that's a model math airplane.

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But we're getting closer.

A definition

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A **mathematical model** is a representation of a real-world situation using mathematical terms.

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We will see: **Math is Everywhere.**

Models underlie many real-world situations

- Countdown clocks: When will the next bus/train arrive?
- Product management: How many items will a store sell?
- Sea rise: How much higher will the oceans be in 50 years?
- Population: How many people will live in NYC in 2030?
- Student demand: How many sections of Math 245 to run?

A justification

Why do we model?

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Why do we model?

$\rightarrow$ observed real-world

$\rightsquigarrow$ behavior

$\rightarrow$

As scientists, we want to understand how the world works.

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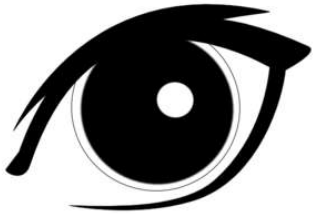

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In this class: We are going to create computer simulations in Python and use the language of mathematics to model the real world.

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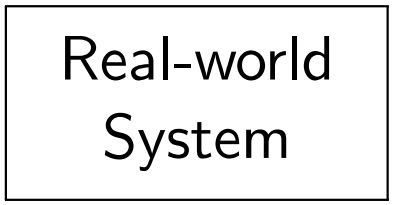

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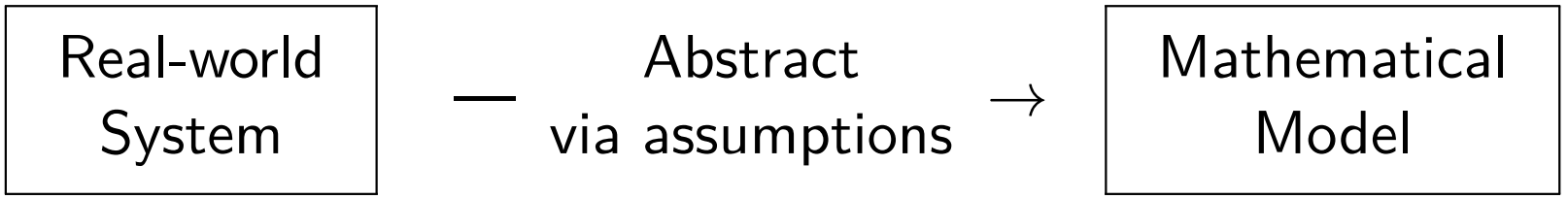

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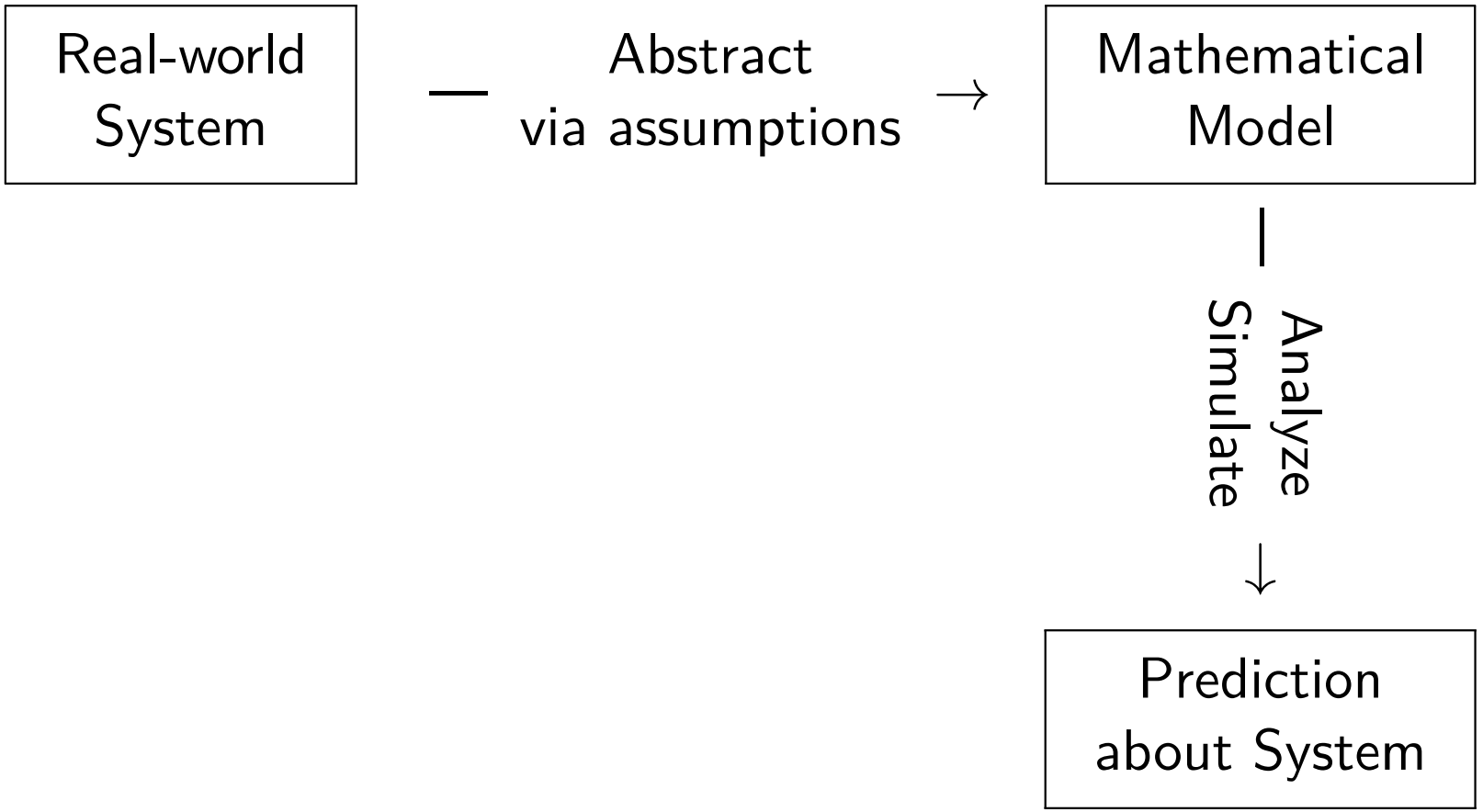

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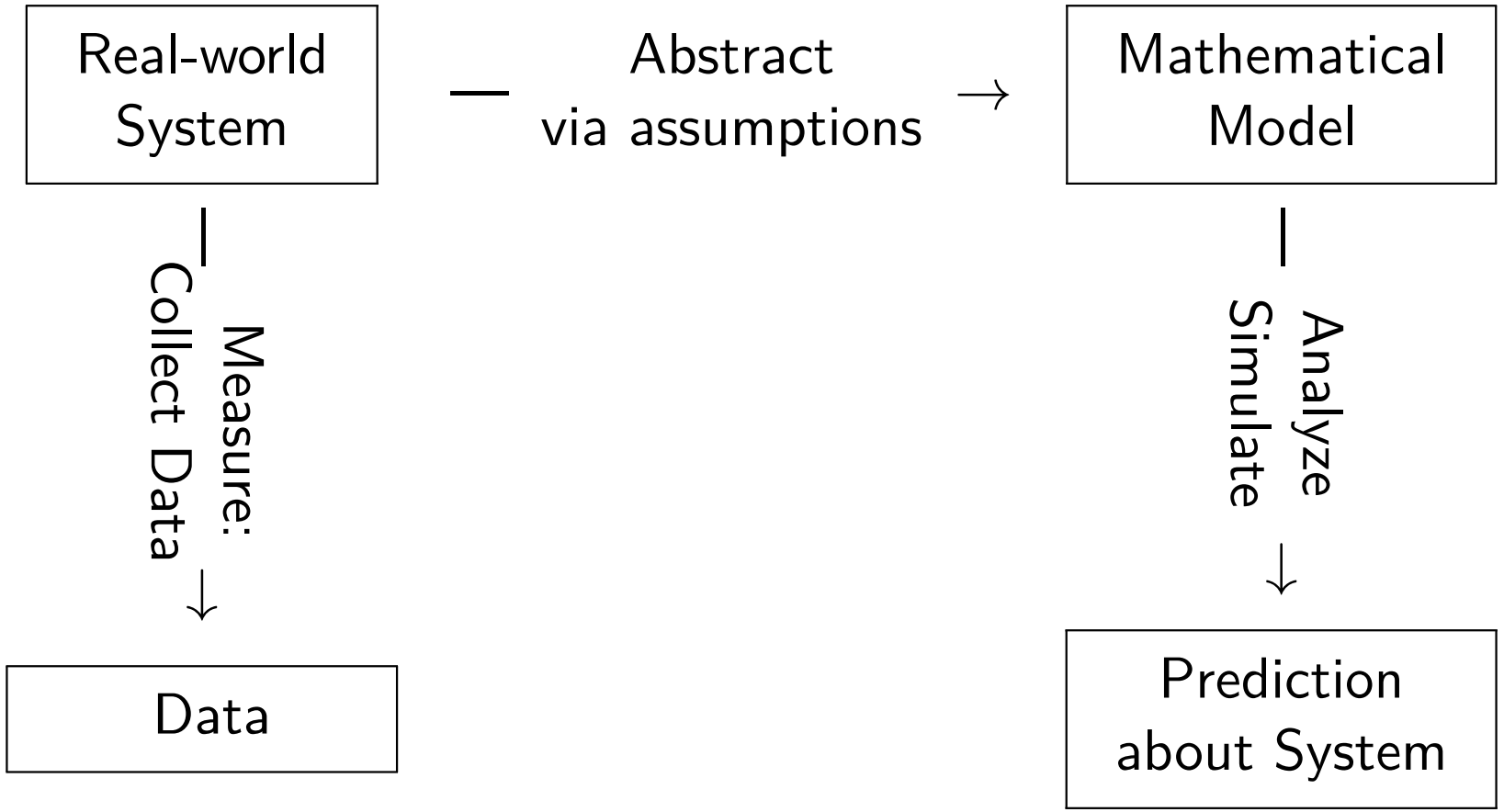

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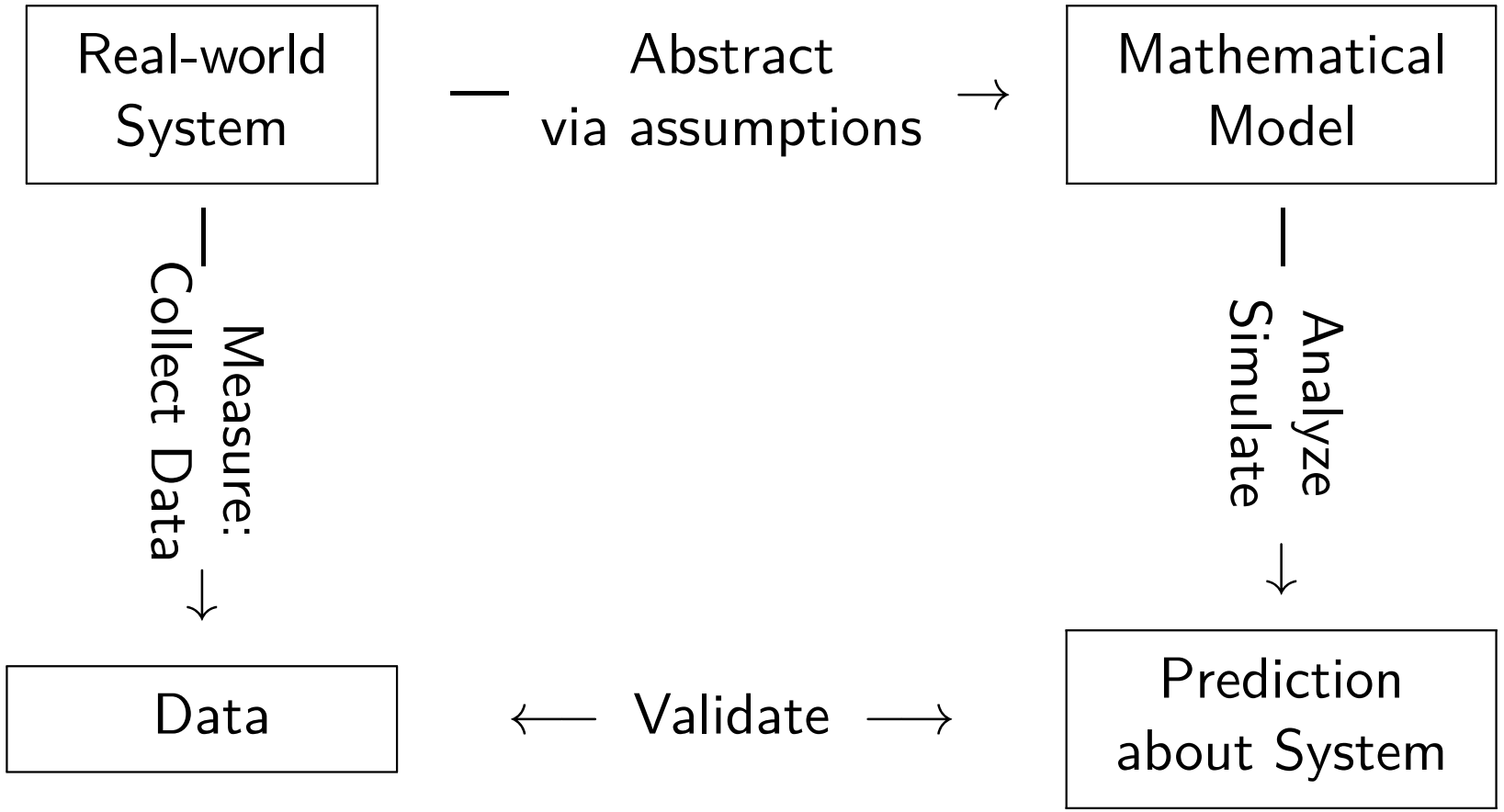

- Identify the most important variables in a real-world system
- Analyze the model / Create a computer simulation
- Collect some data from the real world system
- Validate your model and refine / revise!

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Outside class

- Learning after class
  - Review notes, Do homework, Project work

A normal day in this class

Outside class

- Preparing for class
  - Pre-reading, Answer Daily Question, Prepare questions.

In class

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- Discussion of homework questions, Daily Question.
- New content
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Class Notebook

- Acquire a notebook (50–80 pages) and bring it to class everyday. (Quarantine it from other classes.)
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- First 20–30 pages for homework, the rest for class notes.
- Label each page with the date and label each question.

Question 1-1. (problem statement here)

Answer the question in complete sentences.

(Leave some space for notes from discussion.)

Question 1-2. (problem statement here)

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Everything posted online; first one (many parts) due Wednesday.

Meet the modelers

Group Activity. Get into groups of four-ish people. Take some time to get to know your groupmates:

- Introduce yourself. (your name, where you're from, your major)
- Fill out **the blank side of** your notecard:
  - Write your name. (Stylize if you wish.)
  - Write a few words about your name to help me remember it.
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- *Small talk suggestion:* What kept you busy this past month?

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Thought Question. Brainstorm real-world situations where you interact with a mathematical model. Choose one. How does the mathematical model (or its conclusions) impact your life?

We are going to learn math modeling and python together.

Let's get Started!

- Grab a computer / Bring your own.

- Go to our course webpage:
  `qcpages.qc.cuny.edu/~chanusa/courses/245/20/`
- Find the day's plan $>$ `Content`.
- Head to $>$ `Software`.
- Use your Office365 account to access Azure Notebooks.
- Import the python notebooks from GitHub.
- While they are importing, access Google Classroom.
- Take a minute to answer the Daily Question.

Jupyter notebook advice

Jupyter is the notebook environm't. **Python** is the progr. language.

- **Always work in the `notebooks` directory.**
- Make a **new copy** of the notebook before any modifications.

- Each time we start the server, all previous definitions are lost.
- Use the Python 3 kernel, not the Python 3.6 kernel.

- Jupyter notebooks look linear. They are not.
- Always evaluate the cells in order from top to bottom.

Let's flip a coin!

- We are using the `modsim` package; it must be imported each time we open the notebook.
- `modsim` relies on the `pint` package, so load it first.