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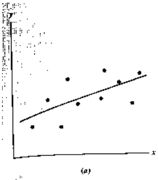

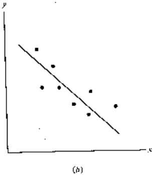

Positive Correlation	**Negative Correlation**
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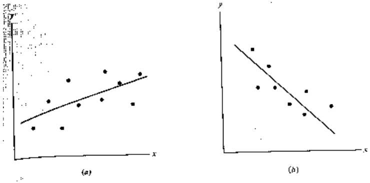

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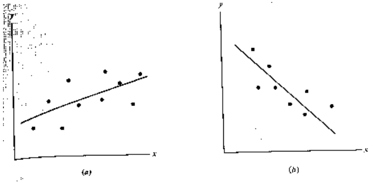

Positive Correlation	**Negative Correlation**
High values of x are associated with high values of y.	High values of x are associated with low values of y.

The **correlation coefficient,** R^2 is a number between 0 and 1.

Values near 1 show strong correlation (data lies almost on a line).
Values near 0 show weak correlation (data doesn't lie on a line).

Calculating the R^2 Statistic

To find R^2, you need data and its best fit *linear* regression.

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- The **error sum of squares**: $SSE = \sum_i \left[y_i - f(x_i)\right]^2$.

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Is my R^2 good? Use a critical value table for R. (Note: not R^2.)
`http://www.gifted.uconn.edu/siegle/research/correlation/corrchrt.htm`

Calculating the R^2 Statistic

Example. (cont'd from notes p. 33) What is R^2 for the data set:
$\{(1.0, 3.6), (2.1, 2.9), (3.5, 2.2), (4.0, 1.7)\}$?

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Example. (cont'd from notes p. 33) What is R^2 for the data set:
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You first need the regression line: $f(x) = -0.605027x + 4.20332$.

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$$SSE = \left(3.6 - f(1.0)\right)^2 + \left(2.9 - f(2.1)\right)^2 + \left(2.2 - f(3.5)\right)^2 + \left(1.7 - f(4.0)\right)^2$$

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$$\begin{aligned} SSE &= \big(3.6-f(1.0)\big)^2+\big(2.9-f(2.1)\big)^2+\big(2.2-f(3.5)\big)^2+\big(1.7-f(4.0)\big)^2 \\ &= (.0017)^2 + (-0.033)^2 + (0.114)^2 + (-0.083)^2 = 0.0210 \end{aligned}$$

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First, calculate $\bar{y} = (3.6 + 2.9 + 2.2 + 1.7)/4 = 2.6$

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$$= (1)^2 + (0.3)^2 + (-0.4)^2 + (-0.9)^2 = 2.06$$

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$$= (1)^2 + (0.3)^2 + (-0.4)^2 + (-0.9)^2 = 2.06$$

- Now calculate $R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{0.0210}{2.06} = 1 - .01 = 0.99$.

Another R^2 Calculation

Example. Estimating weight from height.

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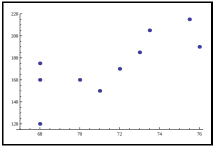

ht.	wt.
68	160
70	160
71	150
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72	170

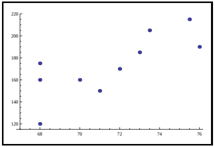
Another R^2 Calculation

Example. Estimating weight from height.

Here is a list of heights and weights for ten students.

We calculate the line of best fit:

$$(\text{weight}) = 7.07(\text{height}) - 333.$$



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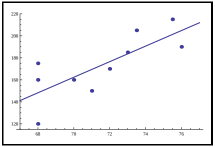

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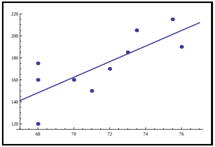

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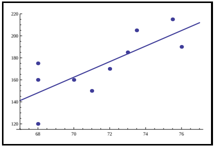

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Now find the correlation coefficient: ($\overline{w} = 173$)

$SSE = \sum_{i=1}^{10} \left[w_i - (7.07\,h_i - 333)\right]^2 \approx 2808$

$SST = \sum_{i=1}^{10} [w_i - 173]^2 = 6910$

So $R^2 = 1 - (2808/6910) = 0.59$

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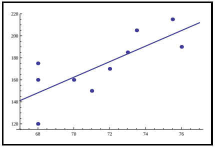

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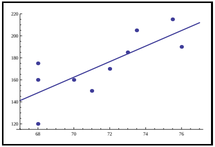

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We can introduce another variable to see if the fit improves.

Multiple Linear Regression

Add waist measurements to the data!

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68	34	160
70	32	160
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68	29	120
68	34	175
76	34	190
73.5	38	205
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Multiple Linear Regression

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We wish to calculate a *linear* relationship such as:

$$(\text{weight}) = a\,(\text{height}) + b\,(\text{waist}) + c.$$

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Do a regression to find the *best-fit plane*:

Use the least-squares criterion. Minimize:

$$SSE = \sum_{(h_i, ws_i, wt_i)} \left[wt_i - (a \cdot h_i + b \cdot ws_i + c)\right]^2.$$

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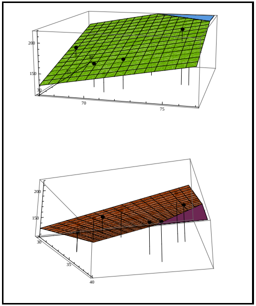
This finds that the best fit plane is (coeff sign)

$$(\text{weight}) = 4.59(\text{height}) + 6.35(\text{waist}) - 368.$$

Multiple Linear Regression

Visually, we might expect a plane to do a better job fitting the points than the line.



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Multiple Linear Regression

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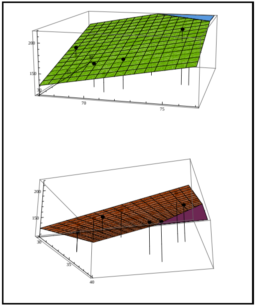
- Now calculate R^2.

Calculate $SSE = \sum_{i=1}^{10}(w_i - f(h_i, ws_i))^2 \approx 955$

SST does not change: (why?)

$\sum_{i=1}^{10}(w_i - 173)^2 = 6910$



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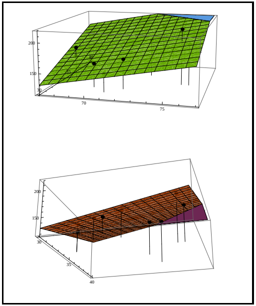

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So $R^2 = 1 - (955/6910) = 0.86$, an excellent correlation.

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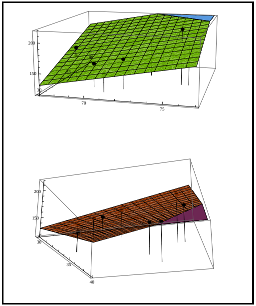

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- When you introduce more variables, SSE can only go down, so R^2 always increases.

Notes about the Correlation Coefficient

Example. Time and Distance (pp. 190)
Data collected to predict driving time from home to school.

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$T =$ driving time
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$T = 1.89M + 8.05$, with an $R^2 = 0.867$.

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- R^2 increases as the number of variables increase.
- This doesn't mean that the fit is better!

Notes about the Correlation Coefficient

Example. Cancer and Fluoridation. (pp. 188–189)

Does fluoride in the water cause cancer?

Notes about the Correlation Coefficient

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Variables:
T = log of years of fluoridation
C = cancer mortality rate

Notes about the Correlation Coefficient

Example. Cancer and Fluoridation. (pp. 188–189)

Does fluoride in the water cause cancer?

Variables:
T = log of years of fluoridation
C = cancer mortality rate

Use a linear regression to find that
$C = \mathbf{27.1}\,T + 181$, with an $R^2 = 0.047$.

Notes about the Correlation Coefficient

Example. Cancer and Fluoridation. (pp. 188–189)

Does fluoride in the water cause cancer?

Variables:

T = log of years of fluoridation A = % of population over 65.

C = cancer mortality rate

Use a linear regression to find that

$C = \mathbf{27.1}\,T + 181$, with an $R^2 = 0.047$.

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- **CAN** determine relative influence of one variable in two models.