The language of optimization

Optimization questions cover a wide variety of situations.
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Optimization questions cover a wide variety of situations.

**Example.** You are given the choice of **one** of the following candies.

<table>
<thead>
<tr>
<th>Snickers bar</th>
<th>Gourmet chocolate square</th>
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<td>Bounty (Coconut+Almond)</td>
</tr>
<tr>
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<td>Tootsie roll lollypop</td>
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**Fact:** You face an optimization problem.

It has a **feasible set**: The set of all valid choices.
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**Fact:** You face an optimization problem.

It has a **feasible set**: The set of all valid choices.

It has an **objective function**: The function we are optimizing over the feasible set.

\[ f : \{ \text{feasible set} \} \rightarrow \{ \text{some measure of goodness} \} \]
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\[ f : \{ \text{feasible set} \} \rightarrow \{ \text{some measure of goodness} \} \]

Our feasible set is ________ and the objective function is ________.
Linear Optimization

We will focus on linear optimization.
Linear Optimization

We will focus on **linear optimization**.

- **Feasible set**: subset of $\mathbb{R}^n$ defined by linear inequalities.
Linear Optimization

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- **Objective function**: linear combination of the input variables.
Linear Optimization

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- **Feasible set:** subset of $\mathbb{R}^n$ defined by linear inequalities.
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Comprehension goals:

- What is a linear program?
- Visualizing linear programs graphically.
- Understanding solutions graphically.
- Solving linear programs using Mathematica.
- Performing sensitivity analysis on linear programs.
Fertilizer example (p.253)

A fertilizer manufacturer uses nitrates and phosphates to make batches of two different kinds of fertilizer.

- Sod-King fertilizer
- Gro-Turf fertilizer
Fertilizer example (p.253)

A fertilizer manufacturer uses nitrates and phosphates to make batches of two different kinds of fertilizer.

- **Sod-King** fertilizer needs 4 phosphates, 18 nitrates.
- **Gro-Turf** fertilizer needs 1 phosphate, 15 nitrates.
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The company has 10 phosphates and 66 nitrates on hand.
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**Question.** How many batches of each should the company make to earn the most profit?
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**Initial thoughts?**

- What do we need to know to make the problem precise?
- What intuition do you have for what the answer should be?
Fertilizer example (p.253)

Translate the problem into mathematics:
We must determine how many batches to make of each.

- Let $x$ represent the number of batches of Sod-King made.
- Let $y$ represent the number of batches of Gro-Turf made.
Fertilizer example (p.253)

Translate the problem into mathematics:
We must determine how many batches to make of each.

- Let $x$ represent the number of batches of Sod-King made.
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What are the constraints on what $x$ and $y$ can be?

- Phosphate constraint:
Fertilizer example (p.253)

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- Phosphate constraint:
- Nitrate constraint:
- Non-negativity constraints:

What are we trying to maximize?

- Profit:
Linear Programs

Maximize $1000x + 500y$

subject to

the constraints:

$4x + y \leq 10$

$18x + 15y \leq 66$

$x \geq 0$

$y \geq 0$
Linear Programs

Maximize $1000x + 500y$

subject to

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$x \geq 0$

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This is a **linear program**, an optimization problem of the form:

Maximize $c_1x_1 + c_2x_2 + \cdots + c_nx_n$ (the **objective function**)

subject to

(the **constraints**):

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$

\vdots

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$
Linear Programs

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\qquad \vdots \qquad \vdots
\quad a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n \leq b_m

Notes about linear programs:

- Constraints may be of the form $\leq$, $=$, or $\geq$. 
Linear Programs

Maximize $c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$ (the objective function)

subject to

(a) $a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \leq b_1$
(b) $a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \leq b_2$
(c) $\vdots$
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Notes about linear programs:

- Constraints may be of the form $\leq$, $=$, or $\geq$.
- The $x_i$ variables are called decision variables.
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- The decision variables can take on any real \( \# \), not only integers.
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- The \( x_i \) variables are called \textbf{decision variables}.
- The decision variables can take on any real \#}, not only integers.
- All constraints and the objective functions are \textit{linear combinations} of the decision variables. (Coefficients are constants.)
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Notes about linear programs:

- Constraints may be of the form $\leq$, $=$, or $\geq$.
- The $x_i$ variables are called decision variables.
- The decision variables can take on any real #, not only integers.
- All constraints and the objective functions are linear combinations of the decision variables. (Coefficients are constants.)
- A linear program in the above form is “easy to solve”.
Fertilizer example, graphically

Maximize \(1000x + 500y\)

subject to

\[4x + y \leq 10\]

\[18x + 15y \leq 66\]

\[x \geq 0\]

\[y \geq 0\]

Let's consider our example graphically.
Fertilizer example, graphically

Maximize \(1000x + 500y\) 
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*Definition:* The set of points \((x, y)\) that satisfy the constraints is called the **feasible region**.
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Let's consider our example graphically.

**Definition:** The set of points $(x, y)$ that satisfy the constraints is called the **feasible region**.

- In general, points of form $(x_1, x_2, \ldots, x_n)$.
- Feasible region always a polytope. (Always has flat sides and is convex.)
- Feasible region may be bounded or unbounded; might be empty.
Fertilizer example, graphically

Maximize $1000x + 500y$

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the constraints:

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★ The solution to the optimization problem will be the point in the feasible region that optimizes the objective function. ★
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Is there a point in the feasible region such that $1000x + 500y = 2000$?
Fertilizer example, graphically

Maximize $1000x + 500y$
subject to $4x + y \leq 10$
the constraints: $18x + 15y \leq 66$
$x \geq 0$
$y \geq 0$

★ The solution to the optimization problem will be the point in the feasible region that optimizes the objective function. ★

Is there a point in the feasible region such that $1000x + 500y = 2000$?
Is there a point in the feasible region such that $1000x + 500y = 4000$?

As we plot these lines of constant objective, we notice that

► They are parallel.
Fertilizer example, graphically

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the constraints:

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Is there a point in the feasible region such that $1000x + 500y = 2000$?

Is there a point in the feasible region such that $1000x + 500y = 4000$?

As we plot these lines of constant objective, we notice that

▸ They are parallel.

▸ If there is a feasible region, at least one line will intersect it.
Fertilizer example, graphically

Maximize $1000x + 500y$

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the constraints:

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As we plot these lines of constant objective, we notice that

- They are parallel.
- If there is a feasible region, at least one line will intersect it.
- As we increase the “constant”, the last place we touch the feasible region is
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Maximize $1000x + 500y$

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$x \geq 0$

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⋆ The solution to the optimization problem will be the point in the feasible region that optimizes the objective function. ⋆

Is there a point in the feasible region such that $1000x + 500y = 2000$?

Is there a point in the feasible region such that $1000x + 500y = 4000$?

As we plot these lines of constant objective, we notice that

► They are parallel.

► If there is a feasible region, at least one line will intersect it.

► As we increase the “constant”, the last place we touch the feasible region is on the boundary, at one or more corners.
Linear Optimization

We have intuited the following theorem.

**Theorem.** The maximum (or minimum) in a linear program either:

1. Occurs at a corner point of the feasible region.
Linear Optimization

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Strategy for solving a linear optimization problem:

0. Determine the decision variables, objective function, and constraints.
1. Draw the feasible region.
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3. Evaluate the objective function at each corner point.
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0. Determine the decision variables, objective function, and constraints.
1. Draw the feasible region.
2. Compute the coordinates of all corner points. (You must think!)
3. Evaluate the objective function at each corner point.
4. Pick out the optimum value.
Solution of fertilizer example

Maximize \( p(x, y) = 1000x + 500y \)

subject to

\[ 4x + y \leq 10 \]
\[ 18x + 15y \leq 66 \]
\[ x \geq 0 \]
\[ y \geq 0 \]

1. Draw the feasible region. (Done!)
Solution of fertilizer example

Maximize \( p(x, y) = 1000x + 500y \)

subject to

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\]

1. Draw the feasible region. (Done!)
2. Compute the coordinates of all corner points.
   - Find the constraints that intersect; solve the associated equalities. (Not all intersections!)
Solution of fertilizer example

Maximize $p(x, y) = 1000x + 500y$

subject to

4x + y ≤ 10

18x + 15y ≤ 66

x ≥ 0

y ≥ 0

1. Draw the feasible region. (Done!)
2. Compute the coordinates of all corner points.
   - Find the constraints that intersect; solve the associated equalities.
   - $x ≥ 0$ and $y ≥ 0$: $(0, 0)$. (Not all intersections!)
Solution of fertilizer example

Maximize \( p(x, y) = 1000x + 500y \)

subject to

\[
\begin{align*}
4x + y &\leq 10 \\
18x + 15y &\leq 66 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

1. Draw the feasible region. (Done!)
2. Compute the coordinates of all corner points.
   - Find the constraints that intersect; solve the associated equalities.
   - \( x \geq 0 \) and \( y \geq 0 \): \((0, 0)\).
   - \( x \geq 0 \) and \( 18x + 15y \leq 66 \): \((0, 22/5)\). (Not all intersections!)
Solution of fertilizer example

Maximize $p(x, y) = 1000x + 500y$

subject to

the constraints:

\begin{align*}
4x + y &\leq 10 \\
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y &\geq 0
\end{align*}

1. Draw the feasible region. (Done!)

2. Compute the coordinates of all corner points.
   - Find the constraints that intersect; solve the associated equalities. (Not all intersections!)
   - $x \geq 0$ and $y \geq 0$: $(0, 0)$.
   - $x \geq 0$ and $18x + 15y \leq 66$: $(0, 22/5)$.
   - $y \geq 0$ and $4x + y \leq 10$: $(5/2, 0)$
Solution of fertilizer example

Maximize $p(x, y) = 1000x + 500y$

subject to

the constraints: $4x + y \leq 10$

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$x \geq 0$

$y \geq 0$

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   - $x \geq 0$ and $18x + 15y \leq 66$: $(0, 22/5)$.
   - $y \geq 0$ and $4x + y \leq 10$: $(5/2, 0)$
   - $18x + 15y \leq 66$ and $4x + y \leq 10$: $(2, 2)$. 

Solution of fertilizer example

Maximize $p(x, y) = 1000x + 500y$

subject to

the constraints:

$4x + y \leq 10$

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$y \geq 0$

1. Draw the feasible region. (Done!)

2. Compute the coordinates of all corner points.
   - Find the constraints that intersect; solve the associated equalities.
     - $x \geq 0$ and $y \geq 0$: $(0, 0)$.
     - $x \geq 0$ and $18x + 15y \leq 66$: $(0, 22/5)$. (Not all intersections!)
     - $y \geq 0$ and $4x + y \leq 10$: $(5/2, 0)$
     - $18x + 15y \leq 66$ and $4x + y \leq 10$: $(2, 2)$.

3. Evaluate the objective function at each corner point.
   - $p(0, 0) = 0$
   - $p(0, 22/5) = 2200$
   - $p(5/2, 0) = 2500$
   - $p(2, 2) = 3000$. 
Solution of fertilizer example

Maximize \( p(x, y) = 1000x + 500y \)

subject to

\[
\begin{align*}
4x + y & \leq 10 \\
18x + 15y & \leq 66 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

1. Draw the feasible region. (Done!)
2. Compute the coordinates of all corner points.
   - Find the constraints that intersect; solve the associated equalities.
     - \( x \geq 0 \) and \( y \geq 0 \): \((0, 0)\).
     - \( x \geq 0 \) and \( 18x + 15y \leq 66 \): \((0, 22/5)\).
     - \( y \geq 0 \) and \( 4x + y \leq 10 \): \((5/2, 0)\)
     - \( 18x + 15y \leq 66 \) and \( 4x + y \leq 10 \): \((2, 2)\).
3. Evaluate the objective function at each corner point.
   - \( p(0, 0) = 0 \) \hspace{1cm} \( p(0, 22/5) = 2200 \)
   - \( p(5/2, 0) = 2500 \) \hspace{1cm} \( p(2, 2) = 3000 \).
4. Pick out the optimum value. \([\text{Max value: } $3000, \text{ occurs at (2,2).}]\)
Using *Mathematica* to solve a linear program

Once you have written your optimization problem as a linear program, you can use *Mathematica* to solve your problem.
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Once you have written your optimization problem as a linear program, you can use *Mathematica* to solve your problem.

Use either the Maximize or Minimize command.

**Syntax:** `Maximize[{obj, constr}, vars]`
Using *Mathematica* to solve a linear program

Once you have written your optimization problem as a linear program, you can use *Mathematica* to solve your problem.

Use either the Maximize or Minimize command.

**Syntax:** \texttt{Maximize[\{obj, constr\},vars]}

- \texttt{obj} is the objective function that you wish to optimize.
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Out[1]: {3000, {x -> 2, y -> 2}}
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**Output:** Optimum value and optimum point.