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Snickers bar	Gourmet chocolate square
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Swedish Fish	Tootsie roll lollypop
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Our feasible set is ________ and the objective function is ________.

Linear Optimization

We will focus on **linear optimization**.

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Comprehension goals:

- What is a linear program?
- Visualizing linear programs graphically.
- Understanding solutions graphically.
- Solving linear programs using *Mathematica*.
- Performing sensitivity analysis on linear programs.

Fertilizer example (p.253)

A fertilizer manufacturer uses nitrates and phosphates to make batches of two different kinds of fertilizer.

- Sod-King fertilizer
- Gro-Turf fertilizer

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Initial thoughts?

- What do we need to know to make the problem precise?
- What intuition do you have for what the answer should be?

Fertilizer example (p.253)

Translate the problem into mathematics:
We must determine how many batches to make of each.

- Let x represent the number of batches of Sod-King made.
- Let y represent the number of batches of Gro-Turf made.

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- Phosphate constraint:

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What are we trying to maximize?

- Profit:

Linear Programs

Maximize $1000x + 500y$

subject to the constraints:

$$\begin{aligned} 4x + y &\le 10 \\ 18x + 15y &\le 66 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

Linear Programs

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This is a **linear program**, an optimization problem of the form:

$$\begin{array}{ll} \text{Maximize } c_1x_1 + c_2x_2 + \cdots + c_nx_n \text{ (the \textbf{objective function})} & \\ \text{subject to} & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\ \text{(the \textbf{constraints}):} & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \\ & \vdots \qquad\qquad\qquad \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \end{array}$$

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- Constraints may be of the form $\leq$, $=$, or $\geq$.

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- The x_i variables are called **decision variables**.
- The decision variables can take on any real #, not only integers.
- All constraints and the objective functions are *linear combinations* of the decision variables. (Coefficients are constants.)
- A linear program in the above form is "easy to solve".

Fertilizer example, graphically

Maximize $1000x + 500y$

subject to the constraints:

$$
\begin{aligned}
4x + y &\leq 10 \\
18x + 15y &\leq 66 \\
x &\geq 0 \\
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\end{aligned}
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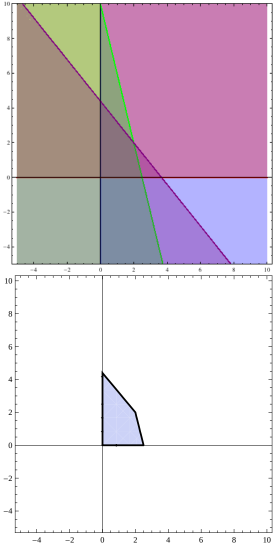
Let's consider our example graphically.



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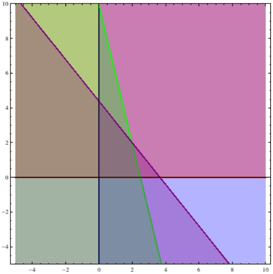

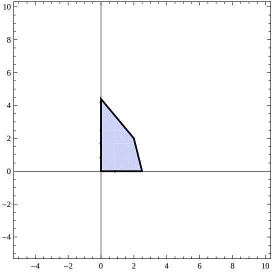

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Definition: The set of points (x, y) that satisfy the constraints is called the **feasible region**.

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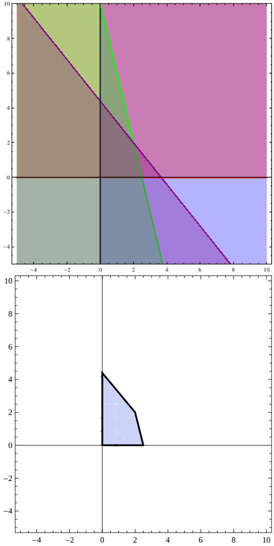
Definition: The set of points (x, y) that satisfy the constraints is called the **feasible region**.

- In general, points of form $(x_1, x_2, \ldots, x_n)$.
- Feasible region always a polytope. (Always has flat sides and is convex.)
- Feasible region may be bounded or unbounded; might be empty.



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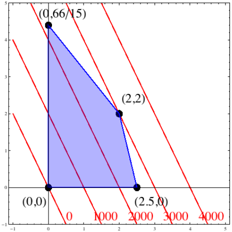

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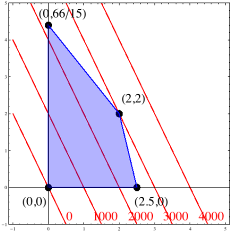

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Is there a point in the feasible region such that $1000x + 500y = 2000$?

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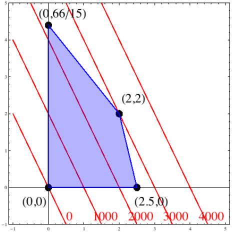

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Is there a point in the feasible region such that $1000x + 500y = 2000$?
Is there a point in the feasible region such that $1000x + 500y = 4000$?

As we plot these lines of **constant objective**, we notice that

- They are parallel.

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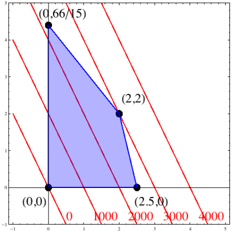

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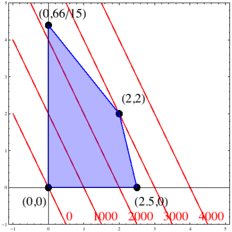

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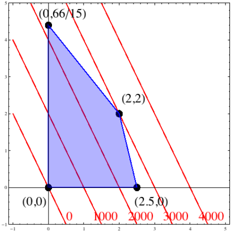

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As we plot these lines of **constant objective**, we notice that

- They are parallel.
- If there is a feasible region, at least one line will intersect it.
- As we increase the "constant", the last place we touch the feasible region is on the boundary, at one or more corners.

Linear Optimization

We have intuited the following theorem.

Theorem. The maximum (or minimum) in a linear program either:

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3. Evaluate the objective function at each corner point.
4. Pick out the optimum value.

Solution of fertilizer example

Maximize $p(x, y) = 1000x + 500y$

subject to the constraints:

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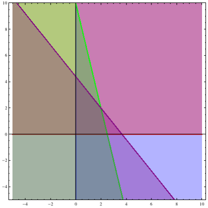

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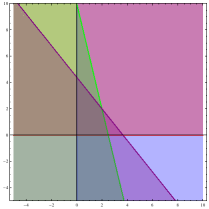

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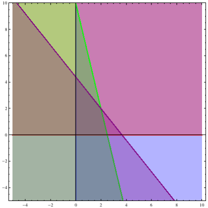

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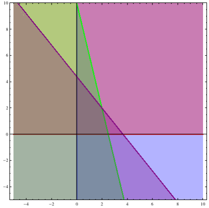

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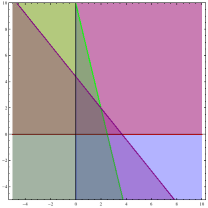

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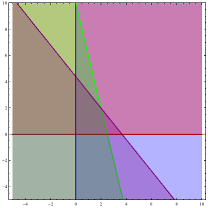

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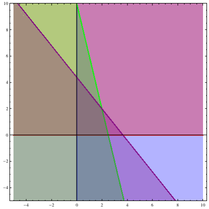
Solution of fertilizer example

$$\text{Maximize } p(x,y) = 1000x + 500y$$

$$\begin{aligned} \text{subject to} \quad & 4x + y \leq 10 \\ \text{the constraints:} \quad & 18x + 15y \leq 66 \\ & x \geq 0 \\ & y \geq 0 \end{aligned}$$



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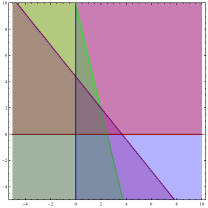

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4. Pick out the optimum value. [Max value: \$3000, occurs at (2,2).]

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Output: Optimum value and optimum point.