Resource Allocation (p. 254)

Translate to a linear program:
(Do not solve)

A farmer allocates 18 acres of land for grapes, potatoes, and lettuce.

Each crop has differ't requirements:

Plot Type	Hours /acre	Capital /acre	Profit /acre
Grapes	9	54	60
Potatoes	9	26	45
Lettuce	3	27	30

She has \$540 and 48 hrs of labor.

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What are the constraints?

What equalities, inequalities must be satisfied by the decision variables? (corresponding to what?)

Diet Problem (p. 255)

Translate to a l.p.:

Create a nutritious soup from two pre-made stocks. Available:

- An onion stock (3¢/oz). 5g protein and 10mg iron
- A chicken stock (2¢/oz). 7g protein and 4mg iron

NCAA requires at least 35g protein and 40mg of iron.

How many oz of each should be included to meet or exceed the requirements at least cost?

Sketch the feasible region.

Find the corners of the region.

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Determine new prices for the un-mixed stocks that would change the optimal solution.
[*Hint: feasible region doesn't change.*]

Integer Programming

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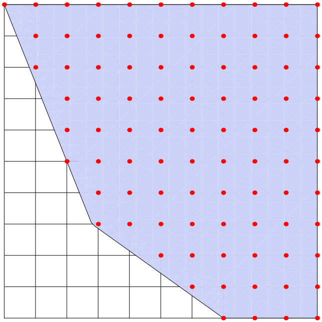
Definition: A linear program with the added restriction that the decision variables must be integers is called an **integer program**.



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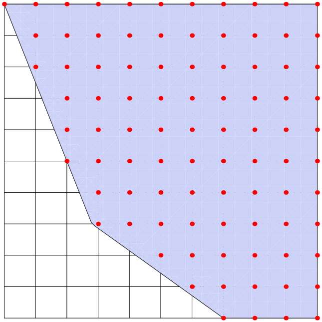
One approach to solving an integer program:

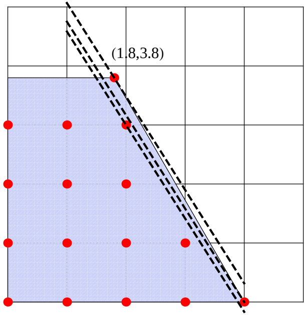
- Solve the linear program by ignoring the integrality constraints
- Take the solution and round the real-valued solution to the nearest *feasible* integer coordinates.



Integer Programming

However, problems may arise if we use this method. (p. 276)

$$\begin{aligned} \text{Maximize } & 3.8x + 2.4y \\ \text{subject to: } & 3.8x + 2.2 \leq 15.2 \\ & y \leq 3.8 \\ & x, y \geq 0 \end{aligned}$$



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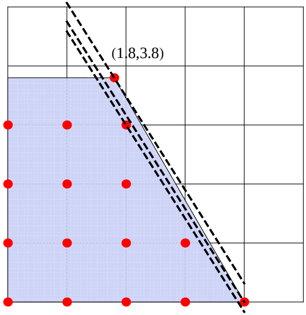

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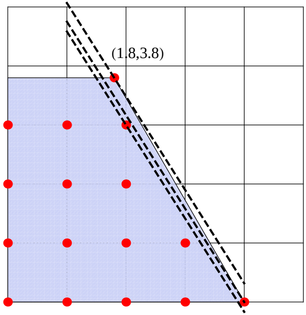

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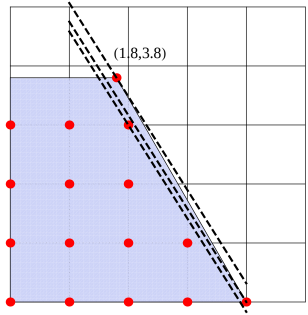

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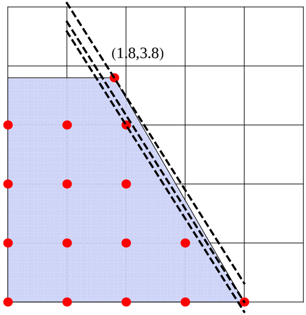

Solving integer programs is **much harder** than solving linear programs.

We still are able to use *Mathematica*:

```
In[1]: Maximize[{3.8x+2.4y, x>=0 && 0<=y<=3.8 &&
                    3.8x+2.2y <= 15.2}, {x, y}, Integers]
Out[1]: {15.2, {x -> 4, y -> 0}}
```

Application: 0-1 variables

Example. Knapsack Problem (p. 277)

A total of m items with fixed weights a_1, a_2, ..., a_m are to be packed in a knapsack. The total weight cannot exceed a fixed weight, b. The objective is to pack as many items as possible.

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Formulation. Create one decision variable x_i for each item i, where we designate that

$$x_i = \left\{ \begin{array}{ll} 1 & \text{if item } i \text{ is to be packed.} \\ 0 & \text{if item } i \text{ is not to be packed.} \end{array} \right\}.$$

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Now: What are we trying to maximize? What are our constraints? IP

Application: 0-1 variables

Example. Assignment Problem (p. 258)

Suppose 3 people P_1, P_2, P_3 are being considered for 3 jobs, J_1, J_2, J_3.

Also suppose that the company knows how much money each person will make the company daily completing each job. (Define it to be a_{ij}.)

	P_1	P_2	P_3
J_1	15	13	12
J_2	11	3	9
J_3	10	5	7

How to assign people to jobs in order to make the most profit?

Application: 0-1 variables

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Goal: Maximize $\sum_{i,j} a_{ij} x_{ij}$ subject to $x_{ij} = 0$ or 1 and

Job constraints: $x_{11} + x_{12} + x_{13} = 1$, $x_{21} + x_{22} + x_{23} = 1$, $x_{31} + x_{32} + x_{33} = 1$.

Ppl constraints: $x_{11} + x_{21} + x_{31} = 1$, $x_{12} + x_{22} + x_{32} = 1$, $x_{13} + x_{23} + x_{33} = 1$.

Carpenter's Problem

Example. Suppose that a carpenter makes tables and bookcases.

- Tables require 20 units of lumber and 5 hours of labor.
- Bookcases require 30 units of lumber and 4 hours of labor.
- She makes a profit of \$25 per table and \$30 per bookcase.
- She has on hand 690 units of lumber and 120 units of labor.

Determine the optimal number of tables and bookcases to build.

Formulation. Let x be the number of tables and y be the number of bookcases she builds in a week. We have the following LP.

$$\begin{array}{rrl} \text{maximize} & z = 25x + 30y & \text{(objective function)} \\ \\ \text{subject to} & 20x + 30y \leq 690 & \text{(lumber constraint)} \\ & 5x + 4y \leq 120 & \text{(labor constraint)} \\ & x, y \geq 0 & \text{(nonnegativity constraints)} \end{array}$$

—Worksheet—

Carpenter's Problem

The worksheet helps us to understand the idea of **sensitivity analysis**.

Given a linear program and its solution,

- How sensitive is the solution to changes in the objective function or the constraints?

The economic interpretation of the results is that the equilibrium cost of lumber is about 71.4 cents per unit and that of labor is $2.14 per hour.

- So if we can buy our lumber for less or hire labor cheaper than that, we should (up to a point)
- And, if we can sell our lumber for more or contract out labor for more, we should (up to a point)

This "point" occurs when the constraint becomes redundant.