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- Inconvenient to experiment with alternate delivery schemes.
  - Disrupt normal service
  - Take surveys of customers
  - Confuse regular customers
- Alternatively, run a computer **simulation**. Write a computer program that models the system of elevators, including:
  - Time of arrival of passengers (a random event)
  - Passenger destination (a random event)
  - Capacity of elevator (fixed by system)
  - Speed of elevator (fixed by system)
  - Current delivery scheme

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Once you have written the computer program,

Verify that the simulation models the current real-world situation

Then, modify various parameters in order to simulate a new delivery scheme.

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- Have the computer keep track of data, such as average wait time, number of stops it takes, longest queue, etc.

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Then, modify various parameters in order to simulate a new delivery scheme.

- How do the data change?
- Is the alternate scheme better or worse?
- Determine how to implement to cause minimal disruption.

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

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- Dealing with probability, so results will always be of the form: "With 95% probability, the wait time will be less than 2 minutes."

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

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The numbers produced by a random number generator are never truly random because they are produced by an algorithm on a deterministic machine.

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  In[1]: CoinFlips = RandomInteger[1,20]
Out[1]: {1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1}
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The sum of this list is the total number of heads tossed.

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  In[2]: Total[CoinFlips]
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Running the commands again will simulate another trial of 20 flips.

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In order to incorporate more complex aspects into the model, use `If` statements and `For` loops.

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Note the double equals sign == and not equals !=.

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- `If[x<0, -x, x]` is _____.
- `If[RandomInteger[] == 1, "Head", "Tail"]`:

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To model this is *Mathematica*, use an `If` statement.

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trial = RandomReal[]
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Alternatively, do this is one step:

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If[RandomReal[] <= 0.075, 1, 0]
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That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

```
trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
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  One time I ran it had 32 successes.

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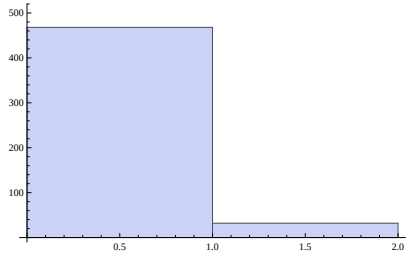
- Alternatively, `Tally[trials]` gives how many times distinct entries appear. Output: `{{0, 468}, {1, 32}}`
- Last, we might want a visualization;
  Use `Histogram[trials]` to get:



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- Now `i` $= 1$, which is still < 4. So '`Print[i]`' is evaluated and `i` is incremented. Similarly for `i` $= 2$ and `i` $= 3$. Now `i` is incremented to 4, which is NOT < 4, and the loop terminates.

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This variable `i` is called a **counter**.
Be careful to name counters wisely! They are defined as variables.

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- `loopCount` is ONLY a counter; it does not change each step's evaluation.

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- After 20 iterations, display '`headCount`' and '`tailCount`'.

```
headCount=0; tailCount=0;
For[loopCount = 1, loopCount <= 20, loopCount++,
  If[RandomInteger[]==1,
      Print["Head"]; headCount++,          ← Notice the ';'
      Print["Tail"]; tailCount++]]         ← Notice the '++'
{headCount, tailCount}
```

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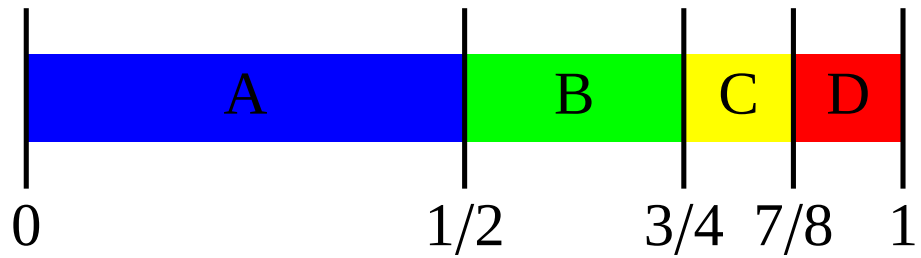

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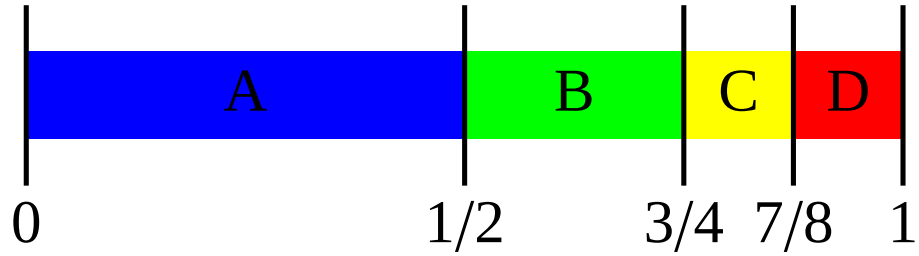

- Reset the counters: 'aCount=bCount=cCount=dCount=0'.
- For loopCount from 1 to 20,
  - Generate a random real number between 0 and 1.
  - If between 0 and 1/2, then output 'A' and aCount++
    if between 1/2 and 3/4, then output 'B' and bCount++
    if between 3/4 and 7/8, then output 'C' and cCount++
    if between 7/8 and 1, then output 'D' and dCount++
- Display 'aCount', 'bCount', 'cCount', and 'dCount'.

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[loopCount = 1, loopCount <= 20, loopCount++,
 roll=RandomReal[];
 If[ 0  <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1 , Print["d"]; dCount++];]
distribution = {aCount, bCount, cCount, dCount}
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- Sample output: (each on its own line)

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- If you are feeling fancy, you can use one `Which` command instead of four `If` commands.

Using Simulation to Calculate Area

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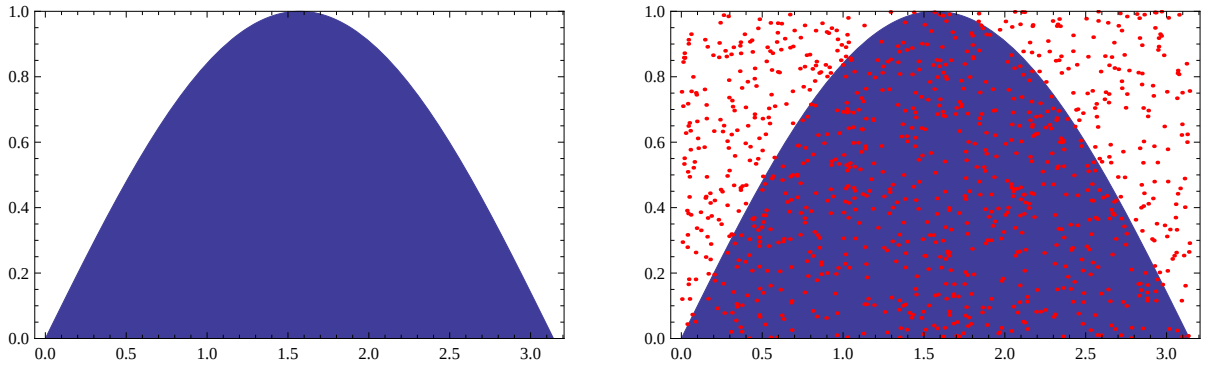
We can approximate this probability by calculating

$$\text{(points falling in region)/(total points chosen)}.$$

Using Simulation to Calculate Area

Example. What is the area under the curve $\sin(x)$ from 0 to π?



Randomly select 100 points from the rectangle $[0, \pi] \times [0, 1]$.

[*Choose a random real between* 0 *and* π *for the* x*-coordinate and a random real between* 0 *and* 1 *for the* y*-coordinate...*]

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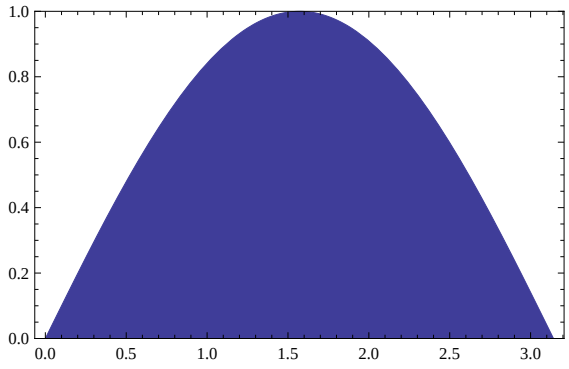

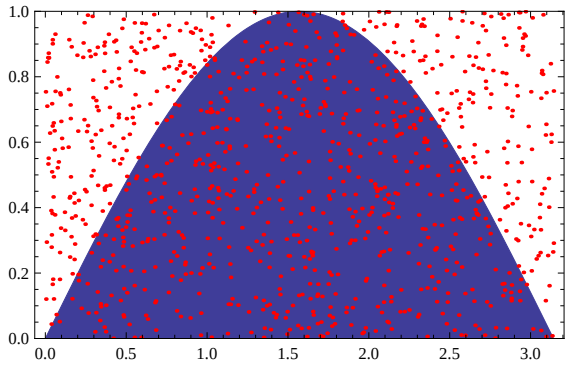

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Here, 63 points fell in the region; we estimate the area to be $\underline{\qquad\qquad}$.

Compare this to the actual value, $\int_{x=0}^{x=\pi} \sin(x)\,dx = [-\cos(x)]_{x=0}^{x=\pi} = 2$