Deterministic versus Probabilistic

Two differing views of modeling:

**Deterministic:** All data is known beforehand

**Probabilistic:** Element of chance is involved
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► Once the system starts, you know exactly what is going to happen.

► **Example.** Predicting the amount of money in a bank account.
  ► If you know the initial deposit, and the interest rate, then:
  ► You can determine the amount in the account after one year.

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**Probabilistic:** Element of chance is involved

▶ You know the likelihood that something will happen, but you don’t know when it will happen.

▶ **Example.** Roll a die until it comes up ‘5’.
  ▶ Know that in each roll, a ‘5’ will come up with probability 1/6.
  ▶ Don’t know exactly when, but we can predict well.
Basic Probability

**Definition:** An **experiment** is any process whose outcome is uncertain.

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Example. The roll of the die . . . [is ‘5’] or [is odd] or [is prime] . . .

Example. $p(E_1) = \text{____}$, $p(E_2) = \text{____}$, $p(E_3) = \text{____}$. 
Determining Probabilities

Three methods for modeling the probability of an occurrence:

- **Relative frequency method:**

- **Equal probability method:**

- **Subjective guess method:**
Determining Probabilities

Three methods for modeling the probability of an occurrence:

- **Relative frequency method:** Repeat an experiment many times; assign as the probability the fraction $\frac{\text{occurrences}}{\# \text{ experiments run}}$.  
  **Example.** Hit a bulls-eye 17 times out of 100; set the probability of hitting a bulls-eye to be $p(\text{bulls-eye}) = 0.17$.

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- **Equal probability method:** Assume all outcomes have equal probability; assign as the probability $\frac{1}{\text{# of possible outcomes}}$. Example. Each side of a dodecahedral die is equally likely to appear; decide to set $p(1) = \frac{1}{12}$.

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Determining Probabilities

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◮ **Subjective guess method:**

   If neither method above applies, give it your best guess.

   **Example.** How likely is it that your friend will come to a party?
Independent Events

**Definition:** Two events are **independent** if the probabilities of occurrence do not depend on one another.

**Example.** Roll a Red die and roll a Blue die.

- Event 1: Blue die rolls a 1. Event 2: Red die rolls a 6. These events are independent.
- Event 1: Blue die rolls a 1. Event 2: Blue die rolls a 6. These events are dependent.
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Example. Pick a card, any card! Shuffle a deck of 52 cards.

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**Example.** You wake up and don’t know what day it is.

- Event 1: Today is a weekday. $E_1 \text{ vs. } E_2$
- Event 2: Today is cloudy. $E_2 \text{ vs. } E_3$
- Event 3: Today is Modeling day. $E_1 \text{ vs. } E_3$
Independent Events

- When events $E_1$ (in $X_1$) and $E_2$ (in $X_2$) are independent events,
  \[ p(E_1 \text{ and } E_2) = p(E_1)p(E_2). \]

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When events $E_1$ (in $X_1$) and $E_2$ (in $X_2$) are independent events,

$$p(E_1 \text{ or } E_2)$$

**Proof:** Venn diagram / rectangle
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- When events $E_1$ (in $X_1$) and $E_2$ (in $X_2$) are independent events, $p(E_1 \text{ or } E_2) = 1 - (1 - P(E_1))(1 - P(E_2))$
  
  $= P(E_1) + P(E_2) - p(E_1)p(E_2)$

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**Example.** What is the probability that you roll a blue 1 OR a red 6?

This does not work with dependent events.
Decision Trees

*Definition:* A **multistage** experiment is one in which each stage is a simpler experiment. They can be represented using a **tree diagram**. Each branch of the tree represents one outcome $x$ of that level’s experiment, and is labeled by $p(x)$. 
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Independent or dependent?

**Example.** Indiana and SF State U. play two soccer games. (p. 382)

Independent or dependent?
Expected value / mean

“Even with the randomness, what do you expect to happen?”
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Suppose that each outcome $x$ in a sample space has a number $r(x)$ attached to it. (Examples: number of pips on a die, amount of money you win on a bet, inches of precipitation falling)
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This function \( r \) is called a random variable.
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**Definition:** The **expected value** or **mean** of a random variable is the sum of the numbers weighted by their probabilities. Mathematically,

$$
\mu = \mathbb{E}[X] = p(x_1)r(x_1) + p(x_2)r(x_2) + \cdots + p(x_n)r(x_n).
$$

**Idea:** With probability $p(x_1)$, there is a contribution of $r(x_1)$, etc.
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Example. How many heads would you expect on average when flipping a biased coin twice?

Example. How many wins do you expect Indiana to have?
Expected value / mean

When two random variables are on two independent experiments, the expected value operation behaves nicely:

\[ E[X + Y] = E[X] + E[Y] \quad \text{and} \quad E[XY] = E[X]E[Y]. \]
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\[
\begin{array}{cccccc}
 b+r & 1 & 2 & 3 & 4 & 5 & 6 \\
 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\end{array}
\]

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\begin{array}{cccccc}
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 4 & 4 & 8 & 12 & 16 & 20 & 24 \\
 5 & 5 & 10 & 15 & 20 & 25 & 30 \\
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\[ E[X + Y] = \]

\[ E[XY] = \]
Component Reliability

Many systems consist of components pieced together.

**Definition:** The **reliability** of a system is its probability of success.

To calculate **system reliability**, first determine how reliable each **component** is; then apply rules from probability.

**Example.** Launch the space shuttle into space with a three-stage rocket.

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\text{Stage 1} \rightarrow \text{Stage 2} \rightarrow \text{Stage 3}
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\[ \text{Stage 1} \rightarrow \text{Stage 2} \rightarrow \text{Stage 3} \]

⋆ In order for the rocket to launch, \underline{\text{Stage 1 success and Stage 2 success and Stage 3 success}}

Let \( R_1 = 90\% \), \( R_2 = 95\% \), \( R_3 = 96\% \) be the reliabilities of Stages 1–3.

\[ p(\text{system success}) = p(S1 \text{ success and } S2 \text{ success and } S3 \text{ success}) \]
Example. Communicating with the space shuttle.
There are two independent methods in which earth can communicate with the space shuttle

- A microwave radio with reliability $R_1 = 0.95$
- An FM radio, with reliability $R_2 = 0.96$.

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Example. Communicating with the space shuttle.
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  ▶ A microwave radio with reliability $R_1 = 0.95$
  ▶ An FM radio, with reliability $R_2 = 0.96$.

☆ In order to be able to communicate with the shuttle,

$$p(\text{system success}) = p(\text{MW radio success or FM radio success})$$