

MATH 634, Spring 2014  
HOMEWORK 14  
due 5:00PM on Wednesday, April 9.

*Background reading: Pearls in Graph Theory, Section 7.1 and 7.2.*

Follow the posted homework guidelines when completing this assignment.

Problems **14D**, **14E**, and **14P** should be typed (or written up) and handed in as class starts on Wednesday 4/9.

- 14D.**
- correctness of an algorithm
  - $M$ -alternating path
  - $M$ -augmenting path
  - stable marriage

**14E.** Use the Hungarian algorithm to solve problem 7.2.2.

**Important:** Use the initial matching  $M = \{(a, 4), (c, 6), (e, 2), (h, 5)\}$ .

**14P.** Prove the correctness of the following algorithm to find a spanning tree of a graph.

*[Prove that the algorithm terminates and that the output of the algorithm is a spanning tree of  $G$ .]*

Input: A connected graph  $G$  with  $n$  vertices.

Preprocess: Label the vertices 1 through  $n$ , color them all white. Let  $T$  be a set of edges, initially empty.

Repeat: For the lowest numbered white vertex  $v$ , order the edges incident with  $v$ . Going through each edge  $e$  from first to last, determine if including  $e$  in  $T$  would create a cycle in  $T$ . If it would not create a cycle, place  $e$  into  $T$  ( $T$  is growing.) If it would create a cycle, do not add  $e$  to  $T$ ; go on to the next edge. Once every incident edge to  $v$  has been checked, color  $v$  black. If all vertices are black, go on to the next step. Otherwise, repeat this step.

Output: Output the graph  $T$ , a spanning tree of  $G$ .