MATH 634, Spring 2014

HOMEWORK 4

to be prepared for presentation at 5:00PM on Monday, February 10.

Background reading: Pearls in Graph Theory, Sections 1.1 through 1.3.

4-1. In parts (a) and (b) below, do not apply Theorem 1.1.2. [*Hint: You will need to find two families of graphs that give an answer for every possible value of n.*]

- Prove that for every even number $n \geq 4$, there exists a graph with n vertices, all of which have degree 3.
- Prove that for every odd number $n \geq 5$, there exists a graph with $n+1$ vertices, n of which have degree 3.

4-2. Prove that no graph has all degrees different. That is, prove that in a degree sequence of a graph, there is at least one repeated number.

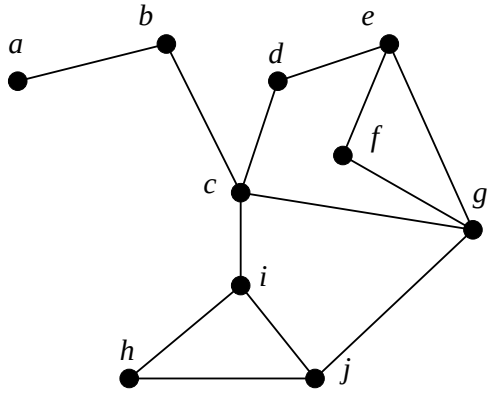
4-3. Explore the proof of Theorem 1.1.2.

The graph below has degree sequence $(\mathcal{S}_1)$ 4 4 3 3 3 2 2 2 2 1. Define $(\mathcal{S}_2)$ to be 3 2 2 2 2 2 2 2 1. Walk through the steps of the proof of Theorem 1.1.2 in the following way.

First, let us choose vertex c from the graph to be vertex S from the proof. Next, assign to each of the remaining vertices $(a - j)$ a name of the form T_i or D_i, just as in the proof.

(a) If you delete vertex S, does the new graph have degree sequence $(\mathcal{S}_2)$?

(b) Use the method in the proof to modify the original graph (possibly applying the algorithm multiple times) so that the resulting graph is such that removing S gives a graph with degree sequence $(\mathcal{S}_2)$.



4-4. (a) Prove that if n is large enough, then the following statement is true:

For all graphs on n vertices, either G or G^c contains a cycle.

(b) For which n does this start to be true?

4-5. Let G be a graph with n vertices and n edges.

(a) Suppose G is connected. How many cycles does G have? Prove it.

(b) Suppose G is **NOT** connected. What can you say about the number of cycles in the graph? Can you determine a formula?

[*Part* (b) *is an exploration question. I want you to explore what happens and write up as much as you can about what you learn.*]