

MATH 634, Spring 2014

HOMEWORK 8

to be prepared for presentation at 5:00PM on Monday, March 3.

Background reading: Pearls in Graph Theory, Sections 2.1 through 3.1.

- 8-1.** (a) Suppose that P and Q are two **maximum** paths in a connected graph G . (That is, no other path in G has longer length.) Prove that P and Q **must** share a common vertex.
- (b) Suppose that P and Q are two **maximal** paths in a connected graph G . (That is, neither is contained in any longer path.) Show that P and Q **might not** share a common vertex.
- 8-2.** (a) Find a graph E which has an Eulerian circuit but no Hamiltonian cycle.
- (b) Find a graph H which has a Hamiltonian cycle but no Eulerian circuit.
- [If either is impossible, prove why you can not find such a graph.]*
- 8-3.** Which of the graphs in Figure 1.2.5 are Hamiltonian?
- 8-4.** (a) Prove that there is no closed knight's tour on the 3×8 grid, without simply citing the theorem from class.
- (b) Find a closed knight's tour on the 3×5 **torus**.
- 8-5.** Is it possible to decompose W_n into two spanning trees? Prove your answer.