MATH 634, Spring 2014
Practice Problems
in preparation for Exam 2 on Monday, May 12, 2014.

The exam covers:

- *Pearls in Graph Theory*, Sections 7.2, 8.1–8.3, 9.1, 9.2, and 10.3.
- Additional topics that are included in the course notes since the first exam, including and not limited to: algorithms, including Gale-Shapley and Ford-Fulkerson.

Below are some questions that practice concepts from the class.

– Book questions: 7.2.3, 8.1.8, 9.1.8, 9.2.3, 9.2.4 (uses a theorem from Section 3.1)

Q1. Let T be a tree and e be any edge of T. Prove that T/e (T contract e) is a tree.

Q2. Show that the planar dual of the octahedron is the cube and vice versa.

Q3. Try to prove the Four Color Theorem by emulating the argument from class using Kempe Chains. What goes wrong?

Q4. When embedding K_6 into the torus, show that the generalization of Euler's formula holds ($p - q + r = 2 - 2g$).

Q5. Prove that the Gale-Shapley algorithm is pet-pessimal when run with the humans proposing. (Hard!)

Q6. The market for kumquats is booming! Five markets (I through V) each have put orders into the five large kumquat distributors (A through E).

- A has 5 kumquats and can deliver to I, II, and III.
- B has 4 kumquats and can deliver to I, III, and IV.
- C has 2 kumquats and can deliver to II and III.
- D has 5 kumquats and can deliver to III and V.
- E has 4 kumquats and can deliver to IV and V.

Market I desires 5 kumquats, II desires 2, III desires 4, IV desires 6, and V desires 3. Is it possible for all the markets to receive their desired quantity of kumquats for the distributors? If so, give a valid transshipment. If not, explain why not.

Q7. Prove that if all the edge costs are different, then there is only one minimum-weight spanning tree.