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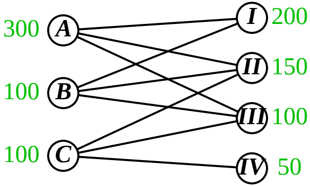

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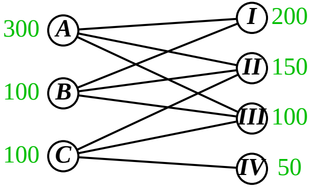

Q: Is there a transshipment that satisfies all the suppliers?

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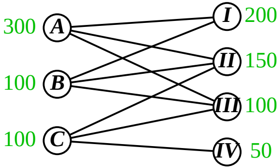

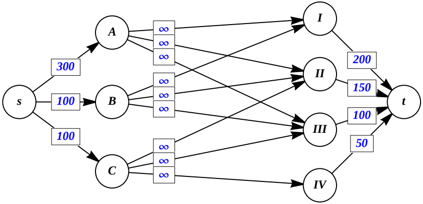

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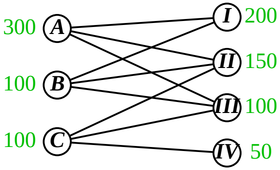

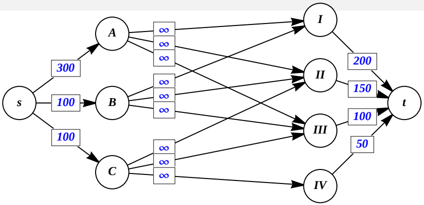

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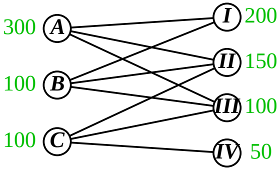

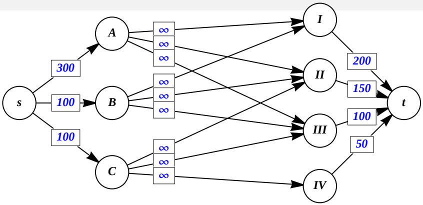

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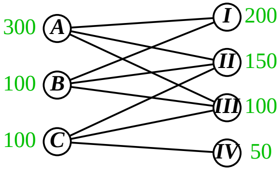

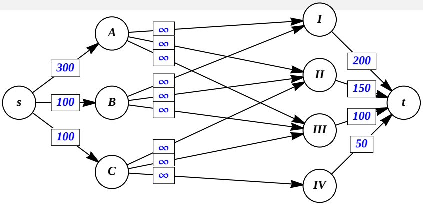

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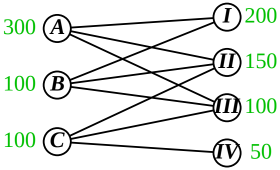

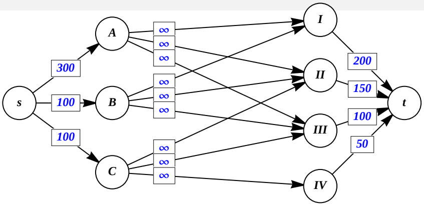

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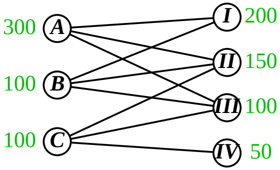

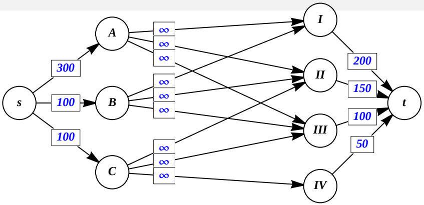

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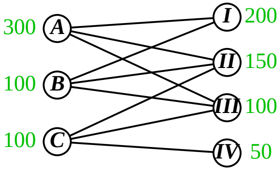

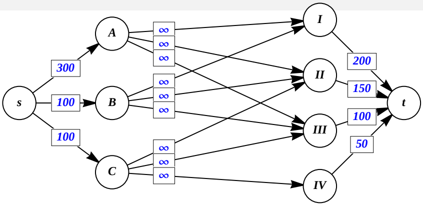

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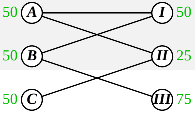
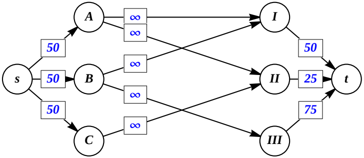
If you are customer-centric, orient the edges from right to left.

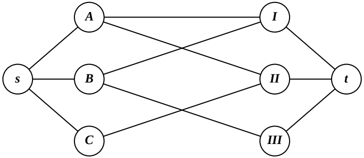
Gives a set of customers who can not be satisfied by their suppliers.

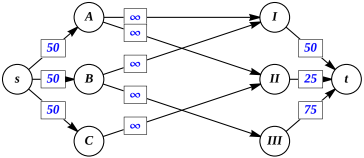
Transshipment Example

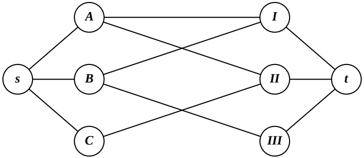


Supplier-centric:









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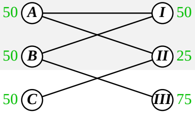

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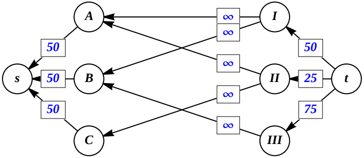

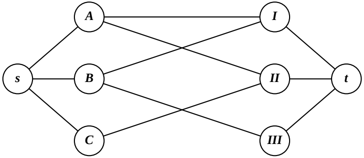

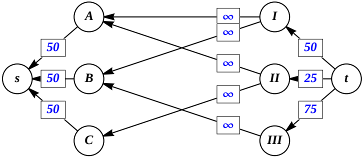

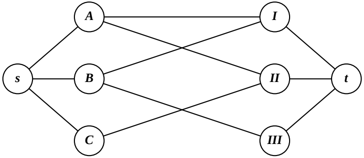

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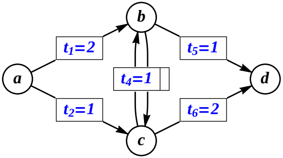

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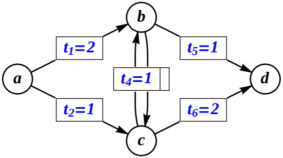
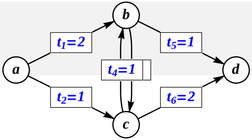


We wish to determine the maximum number of shipments which can make it from city a on day 0 and arrive at city d by day 5.

Dynamic Networks



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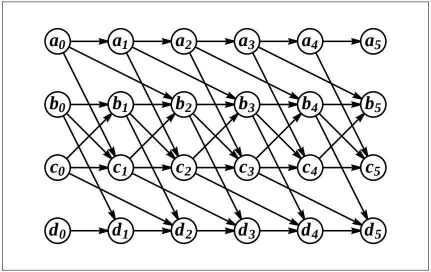

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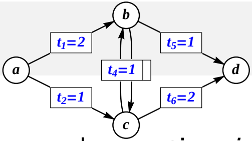

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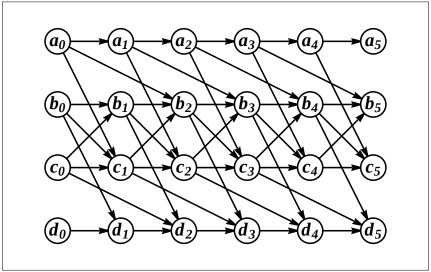

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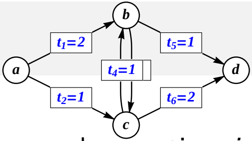

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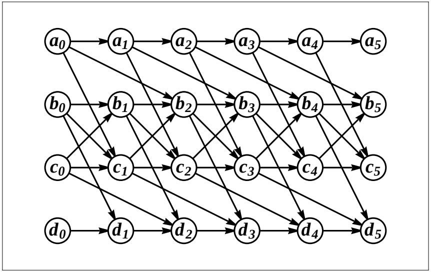

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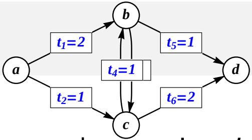

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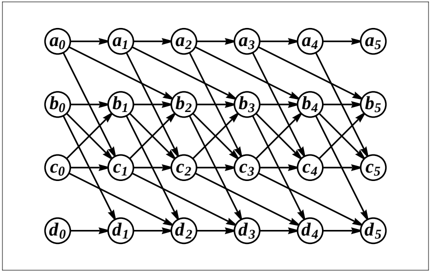

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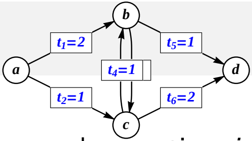

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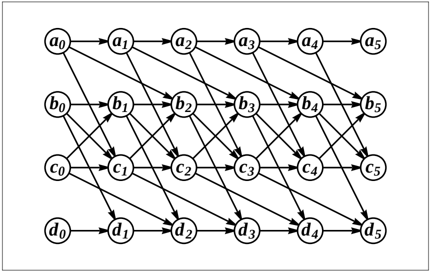

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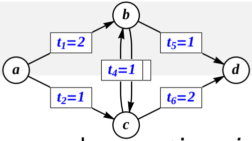

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Example. In the graph below, calculate the max flow from a_0 to d_5.