

# MATH 636, Fall 2014

## HOMEWORK 2

To be prepared for presentation on Tuesday, September 16.

*Background reading: Combinatorics: A Guided Tour, Sections 1.2 through 1.4.*

We will discuss solutions to these questions in class. If you wish to present one of these questions in class, claim it upon arrival. There may also be time for discussion of any questions you may have from class.

Recall that you may submit solutions to these problems for grading, as described on the syllabus.

**2-1.** Answer each of the following jeopardy questions by giving a “real world” situation that could be counted by the given quantity. (*In class, two people can present four each.*)

- Exercises 1.2.1abcd, 2.1.1c, 2.2.1acd

**2-2.** Exercise 1.2.13

**2-3.** In chess, a rook is a piece that can move only vertically and horizontally. Therefore, two rooks attack each other if they are placed in the same row or in the same column. A non-attacking configuration of rooks consists of placing some number of rooks on a chessboard so that no pair of rooks attack each other. Determine the number of non-attacking configurations of five indistinguishable rooks on an  $8 \times 8$  chessboard.

**2-4.** In the following problem, suppose  $k \leq n$ .

- (a) How many functions are there from  $[k]$  to  $[n]$ ?
- (b) How many bijections are there from  $[k]$  to  $[n]$ ?
- (c) How many one-to-one functions are there from  $[k]$  to  $[n]$ ?
- (d) How many onto functions from  $[k]$  to  $[n]$  are not one-to-one?

**2-5.** Create and prove a bijection between two-member subsets of  $\{1, 2, \dots, n, n+1\}$  and all possibilities of placing one pair of parentheses in a string of  $n$  letters. For example, when  $n = 3$ , we see that there are six two-member subsets of  $\{1, 2, 3, 4\}$ :

$$\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \text{ and } \{3, 4\}.$$

and there are six ways to place one pair of parentheses in the word  $abc$ :

$$(a)bc, a(b)c, ab(c), (ab)c, a(bc), \text{ and } (abc).$$

Notice that  $a()bc$  is not valid—there are no letters between the parentheses.

**2-6.** Exercise 1.4.8