MATH 636, Fall 2014
Homework 3
due 5:00pm on Thursday, October 2.

Background reading: Combinatorics: A Guided Tour, Sections 1.3–1.4 and pp. 153–156.

Follow the posted homework guidelines when completing this assignment.
Please **only** consult with your classmates or professor to discuss the problem set.

3-1. (a) Let f be a well-defined function from A to B and let g be a well-defined function from B to A. Suppose that $g(f(a)) = a$ for all $a \in A$. Show that it is not necessarily the case that f is a bijection between A and B.

(b) Let f be a well-defined function from A to B and let g be a well-defined function from B to A. Suppose that $g(f(a)) = a$ for all $a \in A$ and $f(g(b)) = b$ for all $b \in B$. Prove that f is a bijection between A and B.

[*Important: You* **must** *use the definition of bijection.*]

3-2. A **composition** of a positive integer n is a way to write n as a sum of positive integers with no restrictions and where order matters. For example, the compositions of 3 are $1+1+1$, $2+1$, $1+2$, and 3 itself.

Let n be a fixed positive integer. Let $\mathcal{C}$ be the set of compositions of n, and let $\mathcal{B}$ be the set of binary words of length $n-1$. Prove that there is a bijection between $\mathcal{C}$ and $\mathcal{B}$.

3-3. (a) Use the equivalence principle to solve Exercise 1.4.15.

(b) Write a paragraph explaining why we can not use the equivalence principle to count the number of different necklaces where **two** of the n beads are indistinguishable (the same color, for example).

3-4. Use the square-domino interpretation of the Fibonacci numbers to prove that

$$f_{2n} = 1 + \sum_{i=1}^{n} f_{2i-1}.$$

3-5. Let a_n be the maximal number of pieces into which you can cut a circle using n straight lines. **Determine by hand** the first few values of a_n. Use the Online Encyclopedia of Integer Sequences (OEIS) to determine what the formula is for a_n as a function of n.

Once you have found the sequence, there are links right after the first few terms of the sequence. You should look at the graph of the sequence and listen to the sequence. On your homework, write down the 42nd term of the sequence.

Then, via the WebCam link at the bottom of the page, look through a few sequences and write down a sequence that looks interesting (its sequence number, its description and a few first terms) and say why you thought it was interesting.