

MATH 636, Fall 2014

HOMEWORK 5

due 5:00PM on Thursday, October 30, 2014.

Background reading: Combinatorics: A Guided Tour, Sections 3.3 and 3.5, and pp. 114–115. (Ignore any discussion of Exponential Generating Functions.)

Follow the posted homework guidelines when completing this assignment.

There are four questions, each worth four points each.

Please **only** consult with your classmates or professor to discuss the problem set.

I would also suggest that you use some computer algebra software (Mathematica, Sage, Maple, Wolfram Alpha, etc.). When you use the software, **let me know** that you are using software, **what you input** into the software, and **why** the answer you get answers the original question.

5-1. Consider the scenarios in parts (b), (d), (e), and (f) of Exercise 3.3.2 on page 113. For each of these **four** scenarios,

- (i) Find a concise generating function for the situation.
- (ii) Identify the coefficient that you would need to extract to answer the question.
- (iii) Actually extract the coefficient [*in parts (d), (e), and (f)*].

5-2. (a) Algebraically verify the equation

$$\frac{1}{(1-x)(1-x^2)(1-x^3)} = \frac{1}{3(1-x^3)} + \frac{1}{4(1-x^2)} + \frac{1}{4(1-x)^2} + \frac{1}{6(1-x)^3}.$$

- (b) Use the equation from Part (a) to determine an explicit formula for the number of ways to score n points in a basketball game.
- (c) How many ways there are to score 100 points in a basketball game?

5-3. Exercises 3.5.1(b), 3.5.1(d), and 3.6.1(a)

5-4. Use generating functions to prove that for all $n \geq k \geq 1$,

$$\sum_{i=0}^k (-1)^i \binom{n}{i} \binom{n}{k-i} = 0$$