MATH 636, Fall 2014
Homework 6
To be prepared for presentation on Tuesday, November 11.

Background reading: Combinatorics: A Guided Tour, Sections 2.4, 3.4, and 4.4. (Ignore any discussion of Exponential Generating Functions.) Also, the notes on products and compositions of generating functions.

If you wish to present one of these questions in class, claim it upon arrival. (If you have already presented, please let others present this time.)

Recall that you may submit solutions to these problems for grading, as described on the syllabus.

6-1. Exercise 3.3.7 [*Hint: Multiply out some generating functions.*]

6-2. Exercise 3.5.5

6-3. How many ways are there to take a line of n soldiers, break them into non-empty platoons, and choose some (possibly empty) subset of each platoon to be on "night watch"? Give an exact answer, not simply a generating function.

6-4. Suppose you have an unlimited supply of black building blocks of height 1 and an unlimited supply of red, orange, yellow, green, blue, and purple building blocks of height 2. How many ways are there to build a tower of height n? Give an exact answer, not simply a generating function.

[*Hint: Use composition of generating functions.*]

6-5. (a) Determine the generating function for the number of partitions of n such that there are at most two parts of the same size.

[*For example,* 511 *is OK, but* 4111 *is not allowed since* 1 *appears thrice.*]

(b) Determine the generating function for the number of partitions of n such that the parts are all of size equal to a power of two.

[*For example:* 84422 *is OK, but* 744221 *is not because* 7 *is not a power of two.*]

6-6. Solve Exercise 4.4.2. You are given the partition $z_1 + z_2 + \cdots + z_k$ of n, and you now want to investigate the **conjugate partition** $y_1 + y_2 + \cdots$. Try to determine a rule that tells you the value of y_i, the i-th part of the conjugate partition, as some function of the z-values. (Instead of appealing directly to the Ferrers diagram.)

Make sure to explain clearly why your rule works.