Chapter 2
The Basic Neoclassical Model

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Outline

1. Introduction
2. Production
3. Expenditure
4. Equilibrium
5. Fiscal policy

Reading: Chapter 3, Mankiw 6e or 7e

Introduction

- Defining feature of the **Neoclassical model**: all markets function smoothly. Prices adjust to ensure that supply equals demand in all markets.
- Consensus among economists that it is a reasonable assumption in the **long run**. But not in the short run.
- For example, in recessions employment falls for a few quarters. But (so far) the economy has always bounced back.

Learning objectives

1. The economy's demand for labor and capital.
2. Equilibrium in the neoclassical model.
3. The equilibrium in graphs.
4. The equilibrium in equations.
5. The effects of fiscal policy on the economy. Crowding out.

Note: you must learn how to analyze the effects of fiscal policy, **both graphically and with equations**.

Factors of Production

- Labor. We assume a fixed (inelastic) supply of hours of work, $\overline{L}$.
  Example: 1 million workers * 10 h/day * 300 working days = 3,000 million hours of labor
- Capital. Inelastic supply of hours of machine work, $\overline{K}$
  Example: 4 million machines * 10 h/day * 250 working days = 10,000 million hours of capital
- Other production factors (human capital, land and natural resources) hidden in background (Total Factor Productivity).

Technology

- This economy produces a single good Y (output, real GDP) using capital and labor.
- Technology summarized by aggregate production function: $Y = F(K, L)$
- Example 1: Linear production function. $F(K, L) = A(K + bL)$ with $A, b > 0$.
- Example 2: Cobb-Douglas production function. $F(K, L) = AK^{\alpha}L^{1-\alpha}$ with $A, 0 < \alpha < 1$.
- Constant A (Total Factor Productivity) contains the level of technology and any other factor of production (other than capital and labor)

MPL and MPK

- The **marginal product of labor** (MPL) is the increase in production obtained by employing an additional unit of labor, for a fixed amount of capital. Intuitively, $MPL = F(K, L+1) - F(K, L)$.
- Likewise, the **marginal product of capital** is given by $MPK = F(K+1, L) - F(K, L)$.
- Using calculus, MPL is the partial derivative of F with respect to L. MPK is the partial derivative with respect to K.
- Exercise: compute MPL and MPK for the linear and Cobb-Douglas production functions using partial derivatives.

Properties Neoclassical production functions

1. Positive MPL and MPK (only one factor is changing). Intuitively, adding factors always increases output.

$$MPL(K, L) = F(K, L + 1) - F(K, L) > 0$$
$$MPK(K, L) = F(K + 1, L) - F(K, L) > 0$$

2. Diminishing MPL (in L): $MPL(K, L + 1) < MPL(K, L)$.
3. Diminishing MPK (in K): $MPK(K + 1, L) < MPK(K, L)$.
4. MPL increases in K: $MPL(K + 1, L) > MPL(K, L)$.
5. MPK increases in L: $MPK(K, L + 1) > MPK(K, L)$.
6. Constant returns to scale: $F(2K, 2L) = 2F(K, L)$.

Examples

- Using the expressions below check that the C-D production function is neoclassical (for $0 < \alpha < 1$) but the linear production function is not.
- Cobb-Douglas:

$$
\begin{aligned}
MPL &= A(1-\alpha)\left(\frac{K}{L}\right)^{\alpha} \\
MPK &= A\alpha\left(\frac{K}{L}\right)^{\alpha-1}
\end{aligned}
$$

- Linear:

$$
\begin{aligned}
MPL &= Ab \\
MPK &= A
\end{aligned}
$$

The Firm's problem

- One representative firm. Stand-in for all firms in the economy.
- This firm's technology given by Y = F(K,L)
- Goal is to maximize profits: $PY - WL - RK$.
- Firm can choose Y (output), L and K.
- But takes prices as given. Namely, P (the price of its good), W (wage or rental price of labor), and R (rental price of capital) are outside of the firm's control, determined by market forces.

Optimal solution

- Any pair (L,K) such that

$$\begin{aligned} MPL(K,L) &= \frac{W}{P} \\ MPK(K,L) &= \frac{R}{P}. \end{aligned}$$

- **Real wage (W/P)**: units of production that can be purchased with the money received for one hour of work. This is what matters for labor demand (and for labor supply).
- **Real rental rate of capital (R/P)**: cost of renting one unit of capital for one hour in terms of units of production

Proof

1. System of two equations and two unknowns
2. Plot a solution: MPL(K*,L*)= W/P and MPK(K*,L*) = R/P
3. Suppose we are hiring (L*-1) units of labor. Do we want one more unit? As we see in graph, revenue increase (P*MPL(K*,L*-1)) is larger than cost increase (W). So YES!
4. Suppose we are hiring L* units of labor. Do we want one more unit? As we see in graph, revenue increase (P*MPL(K*,L*)) equals cost increase (W). So no reason to hire more labor. We can stop here.
5. Similar argument for capital.

The economy's factor demands

- The **economy's demand for labor** L^d is the amount of labor that maximizes profits at each given real wage, given that all capital in the economy is being used. That is, the solution to $MPL(\overline{K}, L) = W/P$.
- Properties: L^d is a decreasing function of W/P; Increasing in capital stock.
- The **economy's demand for capital** K^d is the solution to $MPK(K, \overline{L}) = R/P$.
- Properties: K^d is a decreasing function of R/P; Increasing in total labor.
- These properties follow from the assumption that F is neoclassical.

Example

- Cobb-Douglas production function for $A = 1$ and $\alpha = \beta = 0.5$.
- Derive the economy's labor demand and capital demand.

Households

- Own the factors of production and the firm. So their income in dollars is $PY = WL + RK + \text{Profits}$.
- Real income (in units of output): $Y = \frac{W}{P}L + \frac{R}{P}K + \frac{\text{Profits}}{P}$
- **Budget constraint**: $Y - T = C + S^P$.

Optimal behavior

- Summarized by the **Consumption function**: $C(Y - T, r)$. Reduced form of intertemporal model.
- Disposable income is Y-T. The increase in consumption following a unit increase in disposable income is called the Marginal Propensity to Consume (MPC).
- The real interest rate is the real return (in units of goods) to savings.
- Assumptions:
  1. Increasing in (Y-T)
  2. $0 < MPC < 1$. Higher disposable income increases both consumption and savings.
  3. Decreasing r. When r goes up, saving is more attractive.
- In textbook $C(Y - T)$ and does not depend on the interest rate.

Investors

- Investors take a one-year loan of size I (units of production) to buy part of the current year's production.
- Use it to build capital (e.g. a new machine) that becomes operative next year.
- They can now rent their capital for R and they repay the loan plus interest, $(1 + r)I$. The new capital good can be used for several years.
- Today's investment increases tomorrow's capital stock: $K_{t+1} = (1 - \delta)K_t + I_t$.
- Optimal behavior of investors summarized by the **Investment function**: $I = I(r, \overline{R_{+1}})$, decreasing in r; increasing in future expected rental rate of capital.

Government

- The government affects demand in two ways: direct purchases (G) and net taxes (T) on households.
- We take as given the government's *fiscal policy*: $(G, T) = (\overline{G}, \overline{T})$.
- Define **government savings** by $S^G = T - G$, which is negative when there is a *budget deficit* and positive when there is a *budget surplus*.
- For now we assume that the government simply consumes G. Later we will consider building highways and high-speed trains (that increase the productivity of the economy).

Definition of Equilibrium

- Given the government's policies, an equilibrium is a vector of prices (W^*, R^*, P^*, r^*) and a vector of quantities $(Y^*, K^*, L^*, C^*, I^*)$ such that:
  1. Given prices, the firm demands factors and supplies output in a way that maximizes its profits.
  2. Given prices, families supply all their factors of production and demand consumption as dictated by their consumption function and the budget constraint.
  3. Given prices, investors' behavior is determined by the investment function.
  4. Prices are such that supply equals demand in all markets: output, factors of production, and loans:

$$
\begin{aligned}
Y^* &= C^* + I^* + \overline{G} \\
K^d &= \overline{K} \\
L^d &= \overline{L} \\
S^P + S^G &= I^*
\end{aligned}
$$

Solution: analytical

1. Market clearing conditions in factor markets implies

$$
\begin{aligned}
Y^* = \overline{Y} &= F(\overline{K}, \overline{L}) \\
\left(\frac{W}{P}\right)^* &= MPL(\overline{K}, \overline{L}) \\
\left(\frac{R}{P}\right)^* &= MPK(\overline{K}, \overline{L})
\end{aligned}
$$

2. The r^* is such that the goods market (and loans market) clears:

$$\overline{Y} = C(\overline{Y} - \overline{T}, r) + I(r) + \overline{G}.$$

3. Finally,

$$
\begin{aligned}
C^* &= C(\overline{Y} - \overline{T}, r) \\
I^* &= I(r^*)
\end{aligned}
$$

Observations

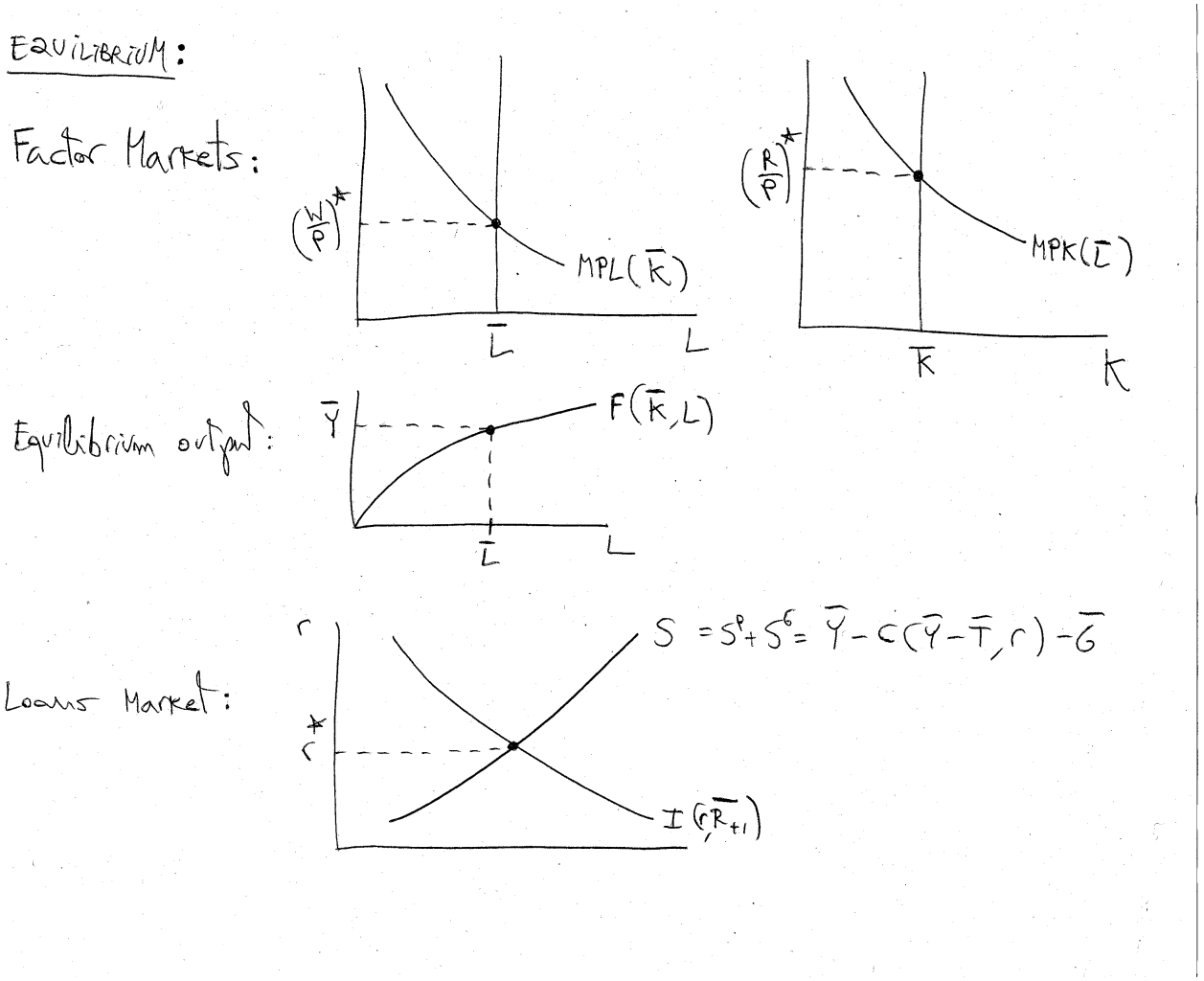
1. The Price level is indeterminate. More on this later.
2. Market clearing in the loans market implies market clearing in the goods market:

$$\begin{aligned} S^P + S^G &= I \\ Y - T - C + T - G &= I \\ Y = C + G + I. \end{aligned}$$

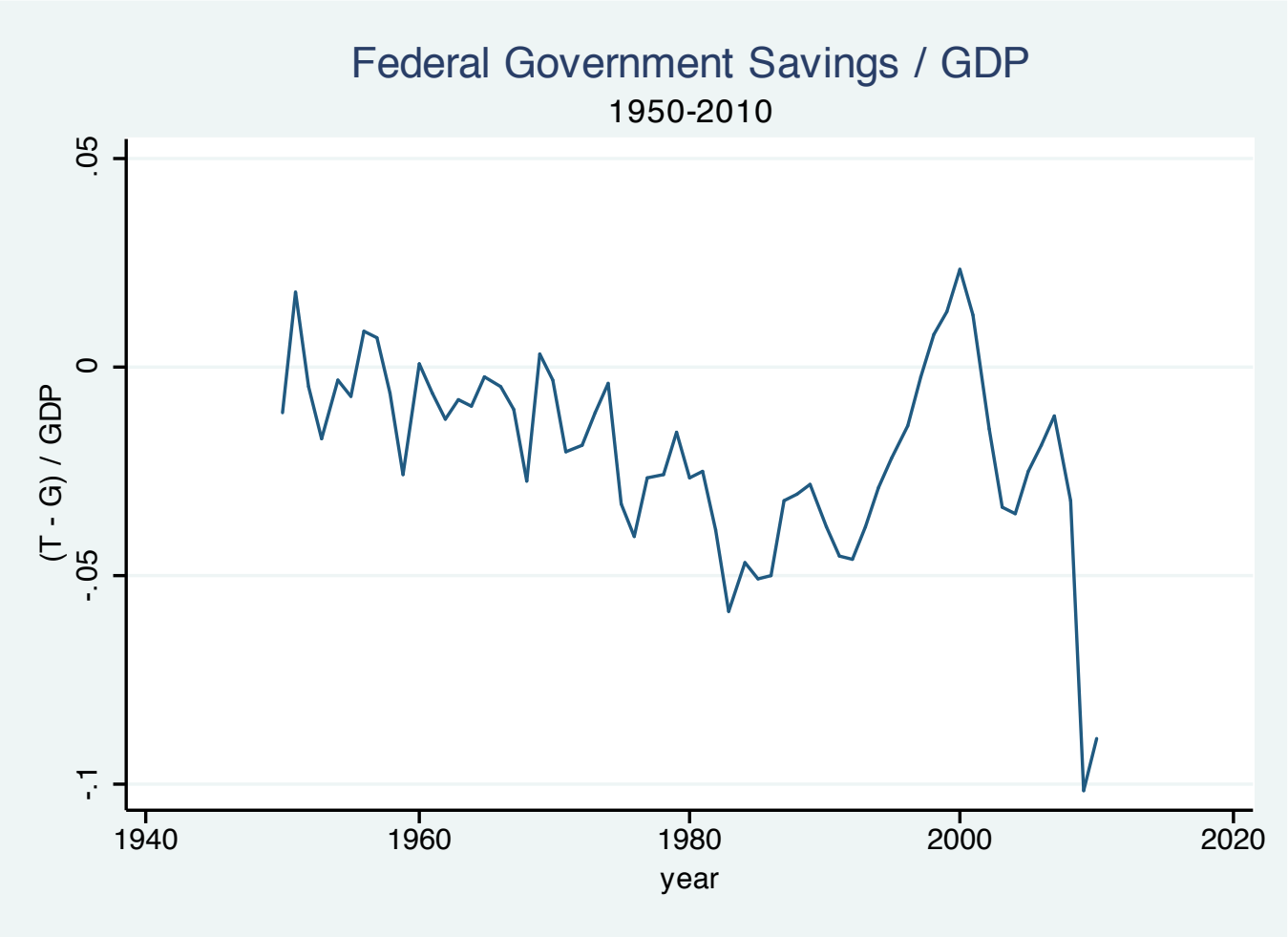
3. Zero economic profits (by Euler's theorem).

$$\begin{aligned} \text{profits} &= P^* F(\overline{K}, \overline{L}) - W^* \overline{L} - R^* \overline{K} \\ &= P^* \left[F(\overline{K}, \overline{L}) - F_L(\overline{K}, \overline{L})\overline{L} - F_K(\overline{K}, \overline{L})\overline{K} \right] = 0 \end{aligned}$$

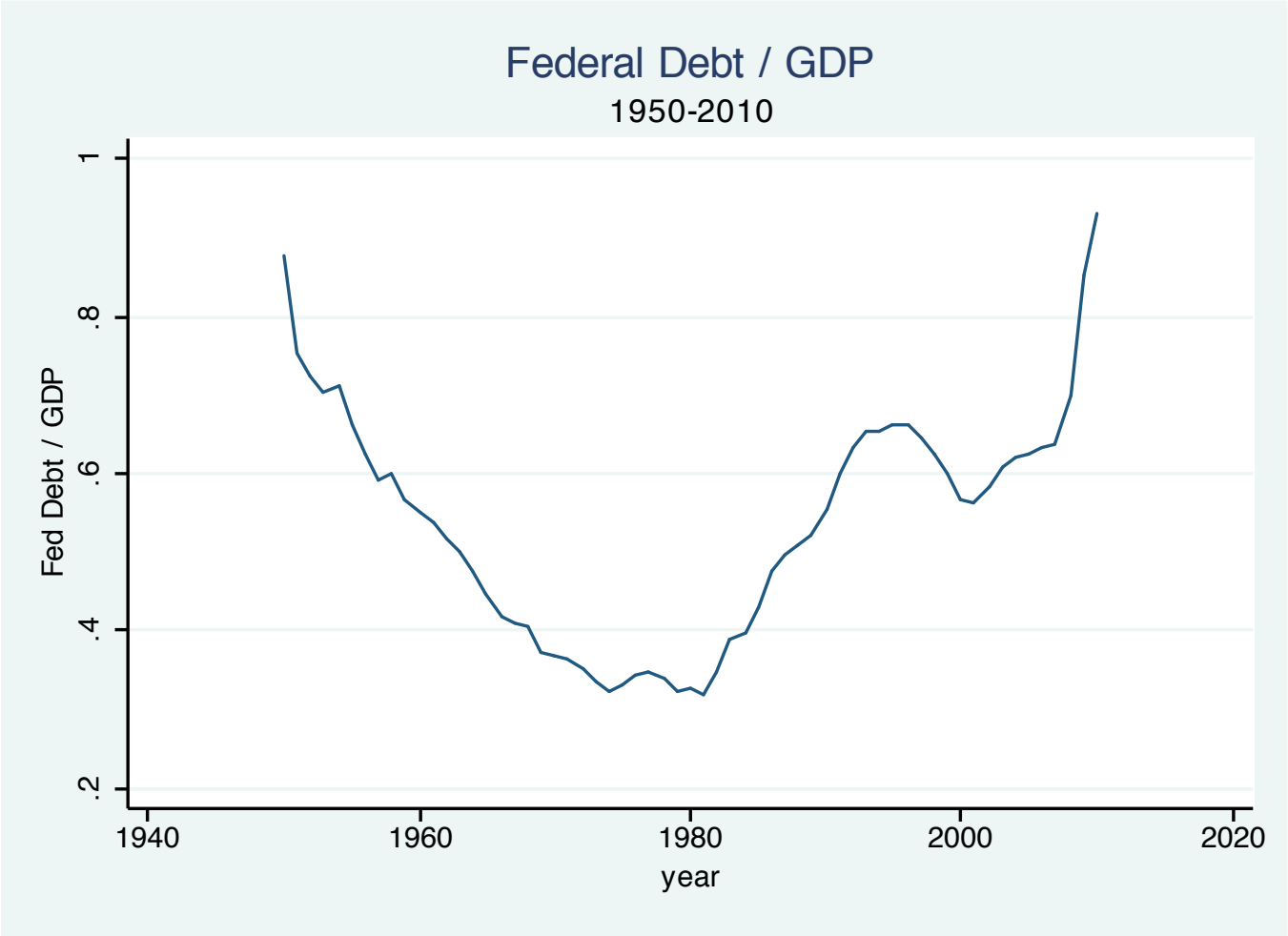
Solution: graphical



Government savings: $T - G$



Debt/GDP


Federal Debt / GDP
1950-2010
Fed Debt / GDP
.2
.4
.6
.8
1
1940
1960
1980
2000
2020
year

Shifts national savings curve

- **National savings curve**: $S(r|\overline{G}, \overline{T}) = \overline{Y} - C(\overline{Y} - \overline{T}, r) - \overline{G}$
- Decreasing function of $\overline{G}$.
- Increasing function of $\overline{T}$ because the MPC is positive.

Changes in fiscal policy

- Do not affect income or employment: $\overline{Y} = F(\overline{K}, \overline{L})$
- But will change the composition of spending.
- Consider policy vectors $(\overline{G}_1, \overline{T}_1)$ and $(\overline{G}_2, \overline{T}_2)$:

$$\begin{aligned}
\overline{Y} &= C_1^* + I_1^* + \overline{G}_1 \\
\overline{Y} &= C_2^* + I_2^* + \overline{G}_2 \\
0 &= -\left(\Delta C^* + \Delta I^* + \Delta \overline{G}\right) \\
\Delta \overline{G} &= -\left(\Delta C^* + \Delta I^*\right)
\end{aligned}$$

- That is, **complete crowding out** of private-sector spending.

An increase in $\overline{G}$

- Use the loans market to show that r^* **rises**.
- Both private investment and consumption fall. In fact, complete crowding out!
- Bad for future income if persistent drop in investment.

Important: Check both graphically and analytically.

Tax cuts: reduction in $\overline{T}$

1. Use the loans market to show that r^* **rises**.
2. Clearly, private investment falls.
3. Consumption rises (not obvious but true - complete crowding out).

Exercise: Suppose consumption does not depend on the real interest rate. Repeat the analysis.

Fiscally neutral changes

- Suppose the government increases its purchases.
- But is required by law to keep a zero budget deficit ($\overline{G} = \overline{T} = \overline{X}$).
- What's the effect on income and on the composition of spending?

$\Delta\overline{G} = \Delta\overline{T} = \Delta\overline{X}$

- Obviously, no effect on income.
- But note that the national savings curve shifts to left!

$$\begin{aligned} S(r|\overline{G},\overline{T}) &= \overline{Y} - C(\overline{Y}-\overline{X}, r) - \overline{X} \\ \frac{\partial S}{\partial \overline{X}} &= \text{MPC} - 1 < 0 \end{aligned}$$

- But shift of savings curve is smaller than when there is solely an increase in $\overline{G}$.
- Both investment and consumption fall.

A numerical exercise

- Consider an economy with

$$
\begin{aligned}
Y &= 1000\sqrt{KL} \\
\overline{G} &= 1000 \\
\overline{T} &= 1000 \\
C &= 250 + 0.75(Y - \overline{T}) \\
I &= 1000 - 50r
\end{aligned}
$$

- Find the equilibrium values for real GDP, consumption, investment, interest rate, and public savings.
- Suppose government purchases increase in 250 units. Find the new equilibrium.

Fiscal policy in the neoclassical world

- Suppose the economy receives a negative shock: A in $Y = AF(K, L)$ temporarily drops.
- Clearly, $\overline{Y}$ will fall. But note that all markets (including the labor market) still clear.
- As we know, standard "expansionary" fiscal policies cannot affect income in the neoclassical model.
- But they will depress private investment. If persistent this will reduce future income!
- Policy prescription: **Laisser-faire**(i.e. do nothing). Wait for the storm to pass.
- Recall that the neoclassical model should be viewed as describing the behavior of the economy in the long run.