QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 131 **Fall 2006**

Instructions: **Answer all questions. Show all work**

1) Given $f(x)=\frac{-1}{2x}$

Find $f'(x)$ using the definition of the derivative.

2) Find the following limits (finite or infinite). Answers may be left in radical form.

(a) $\lim_{x\to 4} \frac{x^2-3x-4}{x^2-16}$ (b) $\lim_{x\to 6} \frac{x-6}{\sqrt{x}-\sqrt{6}}$

(c) $\lim_{x\to -\infty} \frac{5x^3-2x^2+3x-1}{4-3x+6x^2-x^3}$ (d) $\lim_{t\to 2^+} \frac{t^2-2}{t^2-4}$

3) Use your calculator to approximate $\lim_{x\to 0} \frac{9^x-5^x}{x}$. Construct an appropriate table in your answer book and state the limit to three decimal places.

4) Find $\frac{dy}{dx}$ for each of the following functions. Do not simplify.

(a) $y=\left(\frac{x-4}{3x+1}\right)^4+\sqrt{x^3-5x+6}$ (b) $y=\sqrt{\frac{2-e^{4x}}{2+e^{4x}}}$

(c) $y=e^{\sqrt{2x}}\ln\left(5x^2+2\right)$

5) Find an equation of the line which is tangent to the curve $xy^2+2x^2=3y$ at the point $(1,2)$.

6) A computer manufacturer estimates that if x computers can be produced each month, the total cost will be $C(x)=90,000+400x$ dollars. With a unit price $U(x)=1600-3x$ dollars, all computers could be sold.

(a) Find the revenue function.
(b) Estimate, using your graphing calculator, the "break-even" level of production (cost = revenue).
(c) Find the profit function.
(d) How many computers should be produced to maximize the profit?

7) Let $f(x)=x^4-4x^3$. Using calculus, find the intervals of increase/decrease and concavity of the function. Sketch the graph of the function in your answer book. Clearly mark the coordinates of all intercepts, relative minima/maxima, and inflection points, if any, on your graph.

8) The cost of producing q units of a certain product is $c(q)=7500+32q-0.007q^2$ dollars.
(a) Use marginal analysis to estimate the cost of the 1001^{st} item produced.
(b) Calculate the actual cost of the 1001^{st} item.

9) A box is to be constructed with a square base and no top. Material for the sides cost \$2 per square foot while material for the bottom cost \$8 per square foot. If the box costs \$2400, find the dimensions which yield the greatest volume. (Use calculus! Calculator solutions will not be accepted.)

10) The number of bacteria in a Petri dish is growing exponentially.
(a) If there are 200 bacteria in the dish initially and 1,000 after 3 hours, how many bacteria will be in the dish after 10 hours?

(b) At what rate is the bacteria population changing after 2 hours?

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Mathematics 131 **Spring 2006**

Instructions: **Answer all questions.** **Show all work.**

1. Given $f(x)=\sqrt{2x+1}$:

a) Find $f'(x)$ using the definition of the derivative.

b) Using your answer from part (a), find an equation of the tangent line to $f(x)$ at the point where $x=4$.

2. Find the following limits:

a) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}$

b) $\lim\limits_{x\to -2^+}\dfrac{x^2+5x+6}{x^2-4x-12}$

c) $\lim\limits_{x\to 0}\dfrac{e^x}{x}$

d) $\lim\limits_{x\to \infty}\dfrac{7x-x^3}{100x^2+8}$

3. Find $\dfrac{dy}{dx}$ for each of the following functions. DO NOT SIMPLIFY:

a) $y=\dfrac{x^2+1}{e^{5x}}$

b) $y=x^2(\ln x)^3$

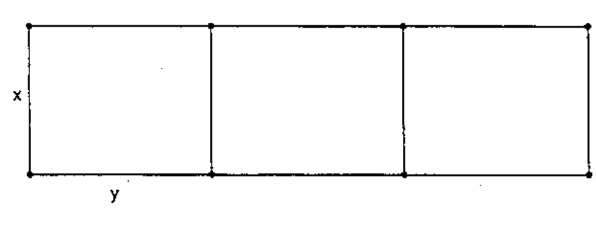
c) Using logarithmic differentiation: $y=\dfrac{(x+3)^4(5x^2+x)^2}{\sqrt{x}}$

d) $xy^2-x^2y+3y^3=12$

4. Given the function $f(x)=\frac{1}{3}x^3-5x^2+16x-3$: Using calculus, find the intervals of increase/decrease and concavity of the function. Sketch the graph of the function in your answer book. Clearly mark the coordinates of all intercepts, relative minima/maxima, and inflection points, if any, on your graph.

5. A farmer wishes to construct three identical adjoining rectangular pens, each with area of 720 square feet. If the outside fencing costs $6 per foot and the inside fencing costs $4 per foot, find x and y to minimize the cost.

Diagram:



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