QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 131 **Fall 2007**

Instructions: **Answer all questions.** **Show all work.**

1. Evaluate the following limits:

a) $\lim\limits_{t\to 9} \dfrac{9-t}{3-\sqrt{t}}$ **b)** $\lim\limits_{t\to 2} \dfrac{t^2+t-6}{t^2-4}$ **c)** $\lim\limits_{x\to +\infty} \dfrac{3x^2-x-2}{5x^2+4x+1}$

2. If $f(x)=\dfrac{2}{\sqrt{x}}$, find $f'(x)$ using the definition of the derivative.

3. **(*i*)** Find $\dfrac{dy}{dx}$: (Do not simplify)

a) $y=\sqrt[4]{\dfrac{x^3+1}{x^3-1}}$ **b)** $y=\dfrac{x^5}{7}-\dfrac{7}{x^5}+\dfrac{\sqrt{x}}{3}-\dfrac{3}{\sqrt[3]{x}}$

(*ii*) Use logarithmic differentiation to find

$f'(x)$ if $f(x)=\dfrac{(x+2)^5}{\sqrt[6]{3x-5}}$

4. a) For what value of the constant c is the function

$$f(x)=\begin{cases} cx+1 & \text{if } x\le 3 \\ cx^2-1 & \text{if } x>3 \end{cases}$$ continuous at $x=3$?

b) Using the intermediate value property, show that there is a root of the equation

$4x^3-6x^2+3x-2=0$ between 1 and 2.

5. The altitude of a triangle is increasing at a rate of $1\,\text{cm}/\text{min}$ while the area of the triangle is increasing at a rate of $2\,\text{cm}^2/\text{min}$. At what rate is the base of the triangle changing when the altitude is $10\,\text{cm}$ and the area is $100\,\text{cm}^2$? ($A=\frac{1}{2}bh$)

6. Find the absolute maximum and absolute minimum values of

$f(x)=x^4-4x^2+2$ on $[-3,2]$.

7. Find an equation of the tangent line to the curve

$y^2=x^3(2-x)$ at $(1,1)$.

(continued on other side)

8. If $1200\,\text{cm}^2$ of material is available to make a box with a square base and an open top, what is the largest possible volume of the box?

9. Let $f(x) = \dfrac{2x^2}{x^2 - 1}$

a) Determine the domain and vertical and horizontal asymptotes.

b) Over what interval(s) is f increasing and over what interval(s) is f decreasing?

c) Over what interval(s) is f concave up and over what interval(s) is f concave down?

d) Use (a), (b) and (c) to sketch the graph of the function. Are there any local maxima/minima? Are there any points of inflection?

10. It is determined that q units of a commodity can be sold when the price is p hundred dollars per unit, where $q(p) = 1{,}000(p+2)e^{-p}$

a) Show that the demand function $q(p)$ decreases as p increases for $p \geq 0$.

b) For what price p is the revenue $R = pq$ maximized? What is the maximum revenue?

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