Mathematics 131   **Spring 2007**

Instructions: Answer all questions. Show all work

1. Find the following limits (finite or infinite):

   (a) $\lim\limits_{x\to-2^+} \dfrac{x-1}{x^2(x+2)}$

   (b) $\lim\limits_{x\to9} \dfrac{\sqrt{x}-3}{x-9}$

   (c) $\lim\limits_{x\to-\infty} \dfrac{3-2x+7x^3-8x^4}{4x^4+11x-5}$

   (d) $\lim\limits_{x\to1} \dfrac{x^2+4x-5}{x^2-1}$

2. Let $f(x)=2x^2$. Find $f'(x)$ using the definition of the derivative.

3. Find $\dfrac{dy}{dx}$ for each of the following functions. <u>Do not simplify</u>:

   (a) $y = \dfrac{(x+3)^5}{\sqrt[6]{2x+7}}$

   (b) $y = e^{x^2}\ln(1+5x^4)$

   (c) Using logarithmic differentiation: $y = \dfrac{e^{-3x}(11x^2-1)^3}{\sqrt[3]{x}}$

4. Find an equation of the line tangent to the curve $x^2y+2y^3 = 3x+2y$ at the point $(0,1)$.

5. Let $C(x) = .25x^2 + 3x + 67$ be the total cost in dollars of manufacturing x units of a certain commodity.

   (a) Find the marginal cost
   (b) Use the marginal cost to estimate the cost of producing the 5th unit.
   (c) Calculate the actual cost of producing the 5th unit.

6. Consider $f(x) = \dfrac{x^2}{x+2}$.

   (a) Find the domain and possible vertical and horizontal asymptotes of f.

   (b) Find the formulas for f' and f'' and use them to determine the intervals of increase or decrease and concavity of f. Use your calculator to sketch the graph of f in an appropriate window and copy what you see into your booklet. Clearly mark the coordinates of all intercepts, relative maxima/minima, and inflection points, if any, on your graph.

7. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 1 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the base of the wall?

8. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs **no** fence along the river. What are the dimensions of the field that has the largest area?

9. (a) Find the absolute minimum and maximum of the function $f(x) = 3x^4 - 16x^3 + 18x^2$ on the interval $[-1,4]$.

   (b) Let $f(x) = (1+x)^{1/x}$. Use your calculator to complete the table below and enter the results into your exam booklet. State the value of $\lim\limits_{x\to0} f(x)$ correct to 3 decimal places using the table values you obtain.

x	-.01	-.001	-.0001	0	.00001	.0001	.001
$f(x)$				**X**			