Department of Mathematics, Queens College
Math 141 Final Exam, Fall 2006

This exam has two parts. You have $2\frac{1}{2}$ hours to answer the questions. You must show all your work in the provided exam book.

<u>PART I</u>: CLEARLY WRITE THE LETTER OF THE CORRECT ANSWER IN YOUR EXAM BOOK. EACH QUESTION HAS 3 POINTS.

1. The domain of the function $f(x) = \sqrt{x} + \sqrt{-x}$ consists of

   (A) all real numbers x (B) all $x \geq 0$ (C) all $x \leq 0$ (D) only $x = 0$

2. $\lim\limits_{x \to -2^+} \dfrac{x^2 - 5}{x + 2}$ is

   (A) $+\infty$ (B) $-\infty$ (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$

3. $\lim\limits_{x \to 0} \dfrac{\sin x}{x + \tan x}$

   (A) is 0 (B) is $\frac{1}{2}$ (C) is 1 (D) does not exist

4. Suppose $f(x) = \begin{cases} x^2 - 1 & \text{for } x \leq 1 \\ 2x - 2 & \text{for } x > 1 \end{cases}$. Then

   (A) f is not continuous at $x = 1$
   (B) f is continuous at $x = 1$ but $f'(1)$ does not exist
   (C) $f'(1)$ exists and is equal to 0
   (D) $f'(1)$ exists and is equal to 2

5. If $f(x) = \sin(x + \cos x)$, then $f'(x)$ is

   (A) $\cos(1 - \sin x)$
   (B) $\cos(x + \cos x)$
   (C) $(1 - \sin x)\cos(x + \cos x)$
   (D) $(1 + \sin x)\sin(x + \cos x)$

6. If $f(x) = \sqrt{1 + \sqrt{x}}$, then $f'(9)$ is

   (A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{48}$

7. The position of a mass suspended from a spring is given by $s(t) = 4t \sin(5t)$, where s is in centimeters and t is in seconds. Its acceleration (in cm/sec^2) at time $t = \pi/10$ is

   (A) -10π (B) -5π (C) 0 (D) 20π

8. An equation of the tangent line to the ellipse $4x^2 + y^2 = 25$ at the point $(2, 3)$ is

   (A) $y = \frac{4}{3}x + \frac{1}{3}$ (B) $y = -\frac{4}{3}x + \frac{17}{3}$ (C) $y = \frac{8}{3}x - \frac{7}{3}$ (D) $y = -\frac{8}{3}x + \frac{25}{3}$

9. If ε is very small, the best linear approximation to $\tan\left(\frac{\pi}{4} + \varepsilon\right)$ is

   (A) 1 (B) $1 + 2\varepsilon$ (C) $1 + \frac{\varepsilon}{2}$ (D) $1 + \varepsilon$

continued on the other side $\longrightarrow$

10. $\lim_{h\to 0} \dfrac{\sqrt[3]{8+h}-2}{h}$ describes the derivative $f'(a)$, where

(A) $f(x) = \sqrt[3]{x}$ and $a = 8$
(B) $f(x) = \sqrt[3]{x}$ and $a = 2$
(C) $f(x) = \sqrt[3]{8+x}$ and $a = 8$
(D) $f(x) = \sqrt[3]{8+x}$ and $a = 2$

11. Suppose f is a differentiable function such that $f(-1) = 1$ and $f(1) = -1$. Then we can find a number c in $(-1, 1)$ such that

(A) $f'(c) = 0$ (B) $f'(c) = -1$ (C) $f'(c) = -2$ (D) $f'(c) = -4$

12. A function f whose derivative is $f'(x) = (x-1)(x^2-1)$ has

(A) a local minimum and a local maximum
(B) a local maximum only
(C) a local minimum and a critical point of neither type
(D) a local maximum and a critical point of neither type

PART II: SOLVE THE FOLLOWING 4 PROBLEMS. MAKE SURE YOU SHOW ALL YOUR WORK.

Problem 1. [10 points] Explain, without using a calculator, why the equation

$$x^3 + 3x - 1 = 0$$

must have a root between 0 and 1. Then use a calculator to find the approximate location of this root. Round your answer to 4 decimal places.

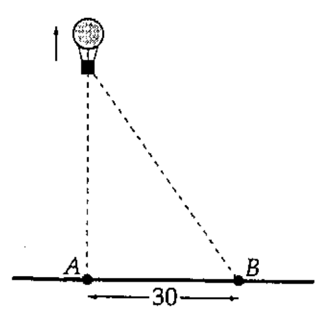
Problem 2. [20 points] Consider the function $f(x) = \dfrac{x+1}{x^2}$.

(i) Find the domain of f and the vertical and horizontal asymptotes of its graph.

(ii) Find the formulas for f' and f'' and use them to determine the intervals of increase/decrease and concavity of f. Make sure you clearly identify the critical point(s) and their type (i.e., local max, local min, neither) as well as the inflection point(s).

(iii) Use your calculator to graph f in an appropriate window and copy what you see in your exam book. Explain why the features of this graph are consistent with your findings in (i) and (ii).

Problem 3. [14 points] A balloon is rising vertically over a point A on the ground at the rate of 15 feet per second. Another point B on the ground is 30 feet from A. At the moment the balloon is 40 feet from A, how fast is its distance from B increasing?



Problem 4. [20 points] A closed cylindrical can is to contain 1000 in^3 of liquid. It costs 1 cent/in^2 to make the side and 3 cents/in^2 to make the top and bottom. What dimensions will minimize the manufacturing cost of this can? What will be the minimum cost? (For a cylinder of base radius r and height h,

volume $= \pi r^2 h$ side area $= 2\pi r h$ top area = bottom area $= \pi r^2$.)