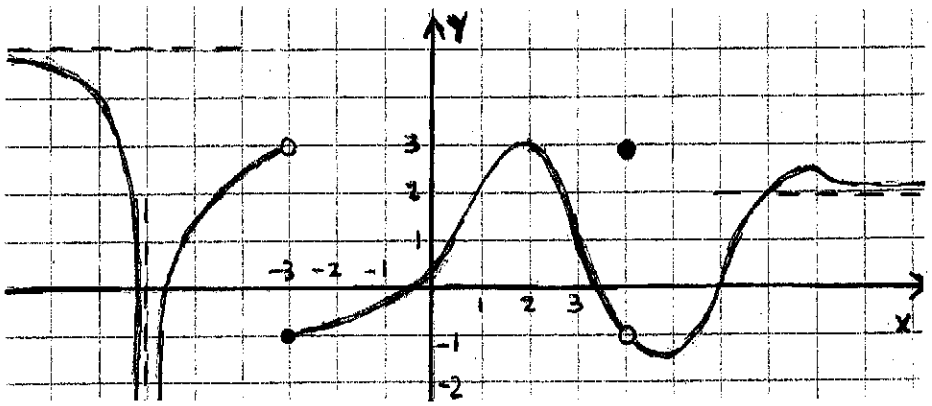
QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ hours

Mathematics 141 Fall 2007

Instructions. Answer each question. Show your work and justify your answers. Partial credit will be awarded for relevant work. When algebraic methods are required, the calculator may be used only to check your answers.

1. Evaluate each of the following, based on the graph of $f(x)$ shown below. If the result is infinite or does not exist, so state.



a. $\lim_{x\to -6} f(x)$ b. $\lim_{x\to -3^-} f(x)$ c. $f(4)$ d. $\lim_{x\to 4} f(x)$

e. $\lim_{x\to -\infty} f(x)$ f. $\lim_{x\to \infty} f(x)$ g. $f'(2)$ h. $f''(3)$

2. Use algebraic methods to determine the following limits. If infinite or non-existent, so state.

a. $\lim_{x\to \frac{1}{2}} \dfrac{4x^2+4x-3}{4x^2-1}$ b. $\lim_{x\to 2+} \dfrac{x^2}{x^2-5x+6}$ c. $\lim_{x\to \infty} \dfrac{9x^2-100x+234}{\sqrt{4x^4+250x-432}}$

3. The function $f(x) = \dfrac{3^x - 2^x}{5^x - 1}$ is not defined when $x = 0$.

a. Use a suitable table of values to estimate $\lim_{x\to 0} f(x)$ to 5 decimal places. [Be sure to copy the table into your answer booklet.]

b. State the formal definition of continuity at a point $x = a$.

c. Can $f(0)$ be defined to make the new function continuous at $x = 0$? Explain.

4. State the formal definition of derivative and use it to work out the derivative of $f(x) = \sqrt{2x+5}$.

5. Find the derivative of each of the following functions. Please make obvious algebraic simplifications.

a. $f(x) = \dfrac{x}{(x+1)^2(x+2)^3}$ b. $g(x) = \sqrt[3]{\sin^2 x - \cos^2 x}$ c. $F(x) = \sec(7x) + \tan(7x)$

(continued on other side)

6. The curve $C: 2y^3 + x^2y = 2x^3 - 10$ defines y as an implicit function $y(x)$ in a neighborhood of the point $(2,1)$ on C.

a. Find an equation for the line tangent to C at $(2,1)$.

b. Let $x = 2.1$ and use the tangent line (linear approximation) to approximate $y(2.1)$.

c. Use your calculator to compute $y(2.1)$ to 4 decimal places directly from the equation for C.

7a. State a theorem which guarantees that the function $f(x) = 2x^3 - 7x - 9\cos x$ takes on an absolute maximum and an absolute minimum value on the interval $[-2, 2]$.

b. Sketch the graph of $f(x)$ in a suitable window. Then use the graph to approximate the coordinates of the maximum point and the coordinates of the minimum point in part (a) to 4 decimal places. Include the sketch in your answer booklet and indicate the window that you are using.

c. Does $f(x)$ take on an absolute maximum on the interval $(-\infty, \infty)$? Justify your answer.

d. The graph has a point of inflection with x in $[-2, 2]$. Approximate its x-coordinate to 4 decimal places. Explain what method you are using to get this level of accuracy.

8a. Find the critical points of $f(x) = 3x^5 - 500x^3 + 100$ by algebraic methods. Indicate whether they are relative maxima or relative minima.

b. Find the points of inflection for the graph of $f(x)$ by algebraic methods.

c. Draw the graph based in the information you found in parts (a) and (b).

d. Use the calculator to graph $f(x)$ in a suitable window that illustrates the results of parts (a) and (b). Include the sketch in your answer booklet and indicate the window that you are using. Comment on any discrepancies between your own graph and the calculator version.

9. Suppose $f(x)$ has a continuous derivative except at $x = 0$ and has ALL the following properties:

i) $f(-3) = 3, \quad f(-2) = 4, \quad f(0) = 1. \quad f(1) = 6, \quad f(3) = 4;$

ii) $\lim_{x\to -\infty} f(x) = 2, \quad \lim_{x\to\infty} f(x) = 1;$

iii) $f'(x)$ is $\begin{cases} \text{negative if} & -2 < x < 0 \text{ or } \quad x > 1, \\ \text{positive if} & \quad x < -2 \text{ or } 0 < x < 1; \end{cases}$

iv) $f''(x)$ is $\begin{cases} \text{negative if} & -3 < x < 0 \text{ or } 0 < x < 3, \\ \text{positive if} & \quad x < -3 \text{ or } \quad x > 3. \end{cases}$

a. Indicate the intervals on which f is increasing or decreasing, respectively.

b. Indicate the intervals on which f is concave up or concave down, respectively.

c. Sketch a possible graph of $f(x)$.

d. Write the equations of all asymptotes.

e. List all local extreme points and indicate whether they are maxima or minima.

f. List all inflection points.

6. The curve $C: \ 2y^3 + x^2y = 2x^3 - 10$ defines y as an implicit function $y(x)$ in a neighborhood of the point $(2,1)$ on C.
   a. Find an equation for the line tangent to C at $(2,1)$.
   b. Let $x = 2.1$ and use the tangent line (linear approximation) to approximate $y(2.1)$.
   c. Use your calculator to compute $y(2.1)$ to 4 decimal places directly from the equation for C.

7a. State a theorem which guarantees that the function $f(x) = 2x^3 - 7x - 9\cos x$ takes on an absolute maximum and an absolute minimum value on the interval $[-2,2]$.
   b. Sketch the graph of $f(x)$ in a suitable window. Then use the graph to approximate the coordinates of the maximum point and the coordinates of the minimum point in part (a) to 4 decimal places. Include the sketch in your answer booklet and indicate the window that you are using.
   c. Does $f(x)$ take on an absolute maximum on the interval $(-\infty, \infty)$? Justify your answer.
   d. The graph has a point of inflection with x in $[-2,2]$. Approximate its x-coordinate to 4 decimal places. Explain what method you are using to get this level of accuracy.

8a. Find the critical points of $f(x) = 3x^5 - 500x^3 + 100$ by algebraic methods. Indicate whether they are relative maxima or relative minima.
   b. Find the points of inflection for the graph of $f(x)$ by algebraic methods.
   c. Draw the graph based in the information you found in parts (a) and (b).
   d. Use the calculator to graph $f(x)$ in a suitable window that illustrates the results of parts (a) and (b). Include the sketch in your answer booklet and indicate the window that you are using. Comment on any discrepancies between your own graph and the calculator version.

9. Suppose $f(x)$ has a continuous derivative except at $x = 0$ and has ALL the following properties:
   i) $f(-3) = 3, \quad f(-2) = 4, \quad f(0) = 1, \quad f(1) = 6, \quad f(3) = 4;$
   ii) $\lim_{x\to -\infty} f(x) = 2, \quad \lim_{x\to\infty} f(x) = 1;$
   iii) $f'(x)$ is $\begin{cases} \text{negative if} & -2 < x < 0 \text{ or } x > 1, \\ \text{positive if} & x < -2 \text{ or } 0 < x < 1; \end{cases}$
   iv) $f''(x)$ is $\begin{cases} \text{negative if} & -3 < x < 0 \text{ or } 0 < x < 3, \\ \text{positive if} & x < -3 \text{ or } x > 3. \end{cases}$
   a. Indicate the intervals on which f is increasing or decreasing, respectively.
   b. Indicate the intervals on which f is concave up or concave down, respectively.
   c. Sketch a possible graph of $f(x)$.
   d. Write the equations of all asymptotes.
   e. List all local extreme points and indicate whether they are maxima or minima.
   f. List all inflection points.