QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 141 **Spring 2007**

Instructions: **Answer all questions.** **Show all work.**

1. a) Without using your calculator, compute the following limits, with $+\infty$ and $-\infty$ as possible values:

i) $\lim_{x\to 3^+} \dfrac{x^2-7x+12}{x^2-6x+9}$ *ii*) $\lim_{x\to 2^-} \dfrac{\frac{1}{4}-\frac{1}{x+2}}{x-2}$

b) Using your calculator, either estimate the following limit to 4 decimal places or determine that it is ∞ or $-\infty$:

$$\lim_{x\to 4^+} \frac{2^x-16}{\sqrt{x}-2}$$

2. a) Let $f(x)=\dfrac{x^2-9}{x-3}$

i) Find $\lim_{x\to 3} f(x)$.

ii) Is f continuous at $x=3$? Explain.

iii) Is f differentiable at $x=3$? Explain.

b) Repeat part a), with $f(x)=\begin{cases}\frac{x^2-9}{x-3}, & \text{if } x\neq 3\\ 5, & \text{if } x\neq 3\end{cases}$

c) Repeat Part a), with $f(x)=\begin{cases}x+3 & \text{if } x\leq 3\\ x^2-5x+12, & \text{if } x>3\end{cases}$

3. Let $f(x)=\sqrt{x+2}$.

a) Using <u>only</u> the definition of the derivative, compute $f'(x)$.

b) Find an equation for the tangent line to the graph of f at the point on the graph whose x coordinate is 7.

4. In each case, find the derivative $\dfrac{dy}{dx}$ (algebraic simplification unnecessary)

a) $y=\left(\dfrac{3}{x^4}+5\right)^6 \tan(7x+8)$ **b)** $y=\dfrac{\sin\left(x^{\frac{4}{3}}\right)}{(\sec(5x))^6}$

c) $x^4y+3y^3=6x+1$

(over)

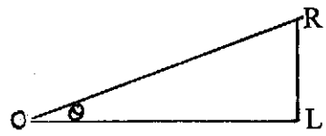
5. Let $f(x)=x^3+2x$.

a) Use the Intermediate Value Theorem to show that there is a positive number c with $f(c)=50$.

b) Show that there can be only one positive number c with $f(c)=50$.
(Hint: consider the sign of $f'(x)$).

c) Use your calculator to find the unique positive number c with $f(c)=50$, correct to five decimal places.

6. An observer [O] is 3 miles from a rocket launch site [L]. The rocket [R] is launched vertically. At the instant when it is 3 miles above the ground, the rocket's speed is 8 miles/minute. How fast is θ changing at this instant if θ is the angle between the line from the observer to the launch site and the line from the observer to the rocket? (For full credit, your answer must be given with the correct units).



7. Let $f(x)=x^4-4x^3+15$.

a) Find the intervals over which f is increasing or decreasing.

b) Find the values of x, if any, at which f has a local minimum or local maximum.

c) Find the intervals over which the graph of f is concave up or concave down.

d) Find the coordinates of the inflection points of the graph of f, if any.

e) Carefully sketch the graph of f.

8. Draw the graph of a function f which has all of the following properties:

$f'(x)>0$ if $x>4$, $f'(x)<0$ if $x<2$ or $2<x<4$

$f''(x)>0$ if $x<0$ or $2<x<5$, $f''(x)<0$ if $0<x<2$ or $x>5$

$\lim_{x\to 2^-} f(x)=-\infty$, $\lim_{x\to 2^+} f(x)=+\infty$, $\lim_{x\to+\infty} f(x)=4$

$f(0)=f(4)=0$, $f(5)=3$

Your graph should include appropriate asymptotes. Inflection points and local extrema, if any, should be labeled.

9. A rectangular banner is to consist of a horizontal white bar below a horizontal red bar. The vertical dimension of the red bar is to be 4 ft and the perimeter of the entire banner is to be 80 ft. The red material costs \$15/sq. ft. and the white material costs \$6/sq. ft. What should the dimensions of the banner be for it to be made as cheaply as possible?