

To get full credit you must show all work

- 1.(7) A box has a square base and a closed top. The volume is 3000 ft^3 . The top costs $\$4/\text{sq ft}$. The bottom costs $\$8/\text{sq ft}$. The sides cost $\$2/\text{sq ft}$. Find the dimensions of the box that minimizes the total cost. Justify that your answer gives the absolute minimum.
- 2.(6) Use the definition of definite integral; i.e., the limit of the Riemann sum, to evaluate $\int_1^2 x^2 dx$.

$$\left[\text{Note: } \sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \right].$$
- 3.(6) Find the exact value of: $\int_5^0 \sqrt{25-x^2} dx$ without using your calculator.
 [Hint: consider the relationship of the definite integral to the area under the curve]
- 4.(20) Find $\frac{dy}{dx}$ for each of the following:
 a) $y = \frac{x^3 \sqrt{x^2+1}}{(x+3)^{3/2}}$ [Hint: Use logarithmic differentiation] b) $y = \sin^{-1}(e^x)$ c) $y = \ln(x^3) + (\ln x)^3$
 d) $y = x^2 + 2^x + x^x + 2^2$ e) $y = \int_{\sin x}^3 \sqrt{3+t^2} dt$
- 5.(28) Evaluate each of the following integrals:
 a) $\int \frac{x dx}{\sqrt{4-x^2}}$ b) $\int \frac{dx}{\sqrt{4-x^2}}$
 c) $\int \frac{1+e^{2x}}{e^x} dx$ d) $\int \frac{e^x}{1+e^{2x}} dx$
- 6.(15) Let R be the region in the plane bounded by the curves $y=x^2$ and $y=x+2$. Set up, but you need not evaluate, the definite integrals for:
 a) the area of R,
 b) the volume generated by rotating R about the x-axis
 c) the volume generated by rotating R about the line $x=-1$
- 7.(8) Let $f(x) = \frac{1}{x+1}$ for $x \geq 0$
 a) Show that f has an inverse. Call it g .
 b) Find $g(x)$
 c) Find $g'(1/2)$ in two ways:
 i. by differentiating the expression for $g(x)$,
 ii. by using the general formula for the derivative of an inverse function.
- 8.(8) Evaluate: a) $\lim_{x \rightarrow \infty} \left(\frac{x}{x-2} \right)^x$ b) $\lim_{x \rightarrow 0} \left(\frac{\sin 3x}{e^x - e^{-x}} \right)$.
- 9.(7) A bacteria culture grows at a rate proportional to its size. After 1 hour the bacteria count was 100 and after 3 hours it was 2500.
 a) What was the initial population of the culture?
 b) Find an expression for the population after t hours.
 c) In what period of time does the population double?