QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 142 **Fall 2006**

Instructions: **Answer all questions.** **Show all work.**

1) Find the following indefinite integrals:

(a) $\int x\tan\left(x^2\right)dx$ (b) $\int \frac{dx}{\sqrt{1-9x^2}}$

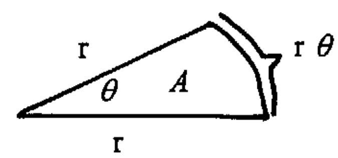
(c) $\int \cos x\, e^{\sin x}\, dx$ (d) $\int \frac{dx}{\sqrt{1-9x}}$

2) Consider the sum $S_n = \sum_{i=1}^{n} \frac{1}{n}\left(2+e^{i/n}\right)$.

(a) Use your calculator to compute S_{20}.

(b) Interpret S_n as a Riemann sum associated with a specific definite integral and thereby evaluate $\lim_{n\to\infty} S_n$

3) A piece of wire is bent into the shape of a sector of a circle as shown in the accompanying figure. If the sector is to enclose an area of π square centimeters, what are the dimensions which <u>minimize</u> the length of the wire?



$\left(\text{area of sector A} = \frac{1}{2}r^2\theta\right)$

4) Let R be the region in the plane bounded by the curves $y=x^2$ and $y=x+12$. Set up, but do <u>not</u> evaluate, definite integrals for:

(a) The area of R

(b) The volume generated by rotating R about the x axis

(c) The volume generated by rotating R about the line $x=10$.

5) Find the derivative dy/dx in each of the following:

(a) $y=x^3+3^x$ **(b)** $y=\sin^{-1}\left(x^2\right)$

(c) $y=x(\ln x)^3$ **(d)** $y=\int_2^x \sqrt{2-\sin t}\; dt$

6) Show $f(x)=4x+\cos x-\sin x$ has an inverse g on <u>any</u> interval. (Do not attempt to find it!) For the interval $[0,3\pi]$, determine the domain and range of g. Given that $\left(\frac{\pi}{4},\pi\right)$ is a point on the graph of f, compute $g'(\pi)$.

7) A continuous function, f, takes on the following values: $f(0)=4$, $f(1)=0$, $f(2)=-6$, $f(3)=-5$, $f(4)=-2$, $f(5)=-1$, $f(6)=1$, $f(7)=5$, $f(8)=-2$, $f(9)=-3$.

(a) Evaluate $\int_1^8 f'(x)\,dx$, **(b)** If $g(x) = \int_1^x f(t)\,dt$ evaluate $g'(5)$.

(over)

8) The fish population in a lake is growing exponentially. If it triples in 10 years, how long does it take to double?

9) Find an equation for the curve that passes through the point $(1,1)$ and such that the slope of the tangent line at (x, y) is $\frac{y^2}{x^3}$.