QUEENS COLLEGE

Math 142 **Final Exam** **DEPARTMENT OF MATHEMATICS** **2 ½ Hours** **Fall 2007**

To get full credit you must show all work

1.(7) A rectangular box, with a top and a bottom, is to be constructed using 48 square feet of material. If the bottom is a rectangle whose length is twice its width, find the dimensions which maximize the volume. Justify that your answer gives an absolute maximum.

2.(6) Use the definition of definite integral; i.e., the limit of the Riemann sum, to evaluate $\int_3^4 x^2 dx$.

$\left(\text{Note: } \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \right).$

3.(6) Find the exact value of: $\int_{-5}^{0} \sqrt{25-x^2}\,dx$ without using your calculator.

(Hint: consider the relationship of the definite integral to the area under the curve.)

4.(18) Find $\frac{dy}{dx}$ for each of the following:

a) $y=\frac{x^3\sqrt{x^2+1}}{(x+5)^{3/2}}$ (use logarithmic differentiation) b) $y=\sin^{-1}(e^x)$

c) $y=\ln(x^3)+(\ln x)^3$ d) $y=x^x+x^3+3^x+3^3$ e) $y=\int_{\sin x}^{3}\sqrt{t^3+3}\,dt$

5.(35) Evaluate, without using your calculator, each of the following integrals:

a) $\int \frac{x}{\sqrt{5-x^2}}dx$ b) $\int_0^{\sqrt{5}/2} \frac{1}{\sqrt{5-x^2}}dx$ (give an exact answer)

c) $\int \frac{1+5^{2x}}{5^x}dx$ d) $\int \frac{e^{2x}}{1+e^{4x}}dx$ e) $\int \frac{e^{4x}}{1+e^{4x}}dx$

6.(15) Let R be the region in the plane bounded by the curves $y=x^2+1$ and $y=x+3$. Set up, but you need **not** evaluate, the definite integrals for

a) the area of R,

b) the volume generated by rotating R about the x-axis

c) the volume generated by rotating R about the line $x=-1$

7.(8) Evaluate: a) : $\lim_{x\to\infty} x^3 e^{-x^2}$ b) $\lim_{x\to\infty}\left(\frac{x}{x+3}\right)^x$

8.(6) Find an equation of the curve whose y-intercept is 5 and the slope of whose tangent line at (x,y) is $3x^2y$.

9.(6) A bacteria culture grows at a rate proportional to its size. If the population doubles in 2 days, how long does it take the population to triple?

10(3) Use your TI83 to evaluate the integral $\int_{\sqrt{3}}^{e}\sqrt{1+x^3}\,dx$, accurate to 5 decimal places.