QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 142 **Spring 2007**

Instructions: Answer all questions. Show all work.
The point values of the questions are shown in parentheses.

1.(7) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y=16-x^2$. Justify that your answer gives an absolute minimum.

2.(6) Use the definition of definite integral (i.e., the limit of the Riemann sum) to evaluate $\int_1^2 x^2 dx$.

$$\left(\text{Note: } \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}\right).$$

3.(6) Find the exact value of: $\int_{-3}^{3} \sqrt{9-x^2}\, dx$ without using your calculator.
(Hint: consider the geometric significance of the definite integral.)

4.(18) Find $\frac{dy}{dx}$ for each of the following:

a) $y=\dfrac{x^2\left(\sqrt[3]{x^2+1}\right)}{(x+3)^{3/2}}$ (use logarithmic differentiation) b) $y=e^{\arcsin x}$

c) $y=\ln\left(x^4\right)+(\ln x)^4$ d) $y=x^x+x^2+2^x+2^2$ e) $y=\int_{x^2}^{3}\sqrt{t^2+5}\,dt$

5.(35) Evaluate each of the following integrals:

a) $\int \frac{x\,dx}{\sqrt{x^2-4}}$ b) $\int \frac{dx}{x\sqrt{x^2-4}}$ c) $\int_0^{\sqrt{3}} \frac{dx}{3+x^2}$ (give an exact answer)

d) $\int \frac{1+e^{2x}}{e^x} dx$ e) $\int \frac{e^x}{1+e^{2x}} dx$

6.(15) Let R be the region in the plane bounded by the curves $y^2=x$ and $y=x-2$. Set up, but you need **not** evaluate, the definite integrals for:

a) the area of R
b) the volume generated by rotating R about the y-axis
c) the volume generated by rotating R about the line $y=-1$

(over)

7.(3) Use your TI83 to evaluate the integral $\int_{\sqrt{2}}^{\pi} \sqrt{1+x^3}\,dx$, accurate to 5 decimal places.

8.(6) Solve the following differential equation for y, if $y(1)=-3$, where $x>0$:

$$\frac{dy}{dx}=\frac{y(1+x)}{x}.$$

9.(6) Evaluate: a) $\lim_{x\to 0}\left(\frac{e^x-x-1}{x^2}\right)$ b) $\lim_{x\to 0}(\cos x)^{2/x^2}$