Queens College

DEPARTMENT OF MATHEMATICS

Final Examination

$2\frac{1}{2}$ Hours

Math 143

FALL ~~Spring~~ 2006

Instructions. Answer all questions. Show your work. You may not use a TI 89 calculator or higher on this exam or any calculator that does indefinite integration.

1. Integrate each of the following.
   - **a.** $\int \tan^{-1} 3x\,dx$
   - **b.** $\int \frac{x^3 - x^2 + 2x + 2}{x^2 + 2}\,dx$
   - **c.** $\int \frac{5x^2 - 1}{(x-2)(x-3)(x+1)}\,dx$
   - **d.** $\int \sec^3 x \tan^3 x\,dx$
   - **e.** $\int \sqrt{9 - x^2}\,dx$
2. Determine if the following series converge or diverge and give reasons.
   - **a.** $\sum_{1}^{\infty} \frac{e^n}{1 + \ln n}$
   - **b.** $\sum_{1}^{\infty} \frac{n^3}{n^5 + n + 1}$
   - **c.** $\sum_{2}^{\infty} \frac{4}{n(\ln n)^3}$
   - **d.** $\sum_{1}^{\infty} \frac{2^n}{3^n + n + 1}$
   - **e.** $\sum_{1}^{\infty} \frac{(n!)^3}{(3n)!}$
3. Find the interval of convergence for the series
$$\sum_{1}^{\infty} \frac{(x+1)^n}{(3^n)5n}.$$
4. Determine if the following integrals converge or diverge.
   - **a.** $\int_{1}^{\infty} xe^{-x^2}\,dx$
   - **b.** $\int_{-1}^{3} \frac{1}{(x-1)^3}\,dx$

5. WITHOUT USING YOUR CALCULATOR, find the limit of each of the following sequences.
   a. $\left\{\frac{n^3-1}{n^3+1}\right\}$
   b. $\left\{\frac{(2n)!}{(2n+3)!}\right\}$
   c. $\left\{n\sin^{-1}\left(\frac{2}{n}\right)\right\}$
   d. $\left\{(1+n)^{5/n}\right\}$

6. Given that $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$, write a power series in x that represents the integral $\int x^4 \sin(x^2)dx$. For which x is this representation valid?

7.
   a. Given the alternating series $\sum_{1}^{\infty} \frac{(-1)^{n+1}n}{(n^2+1)}$, what is the smallest number of terms one must add to be assured that the resulting sum will be within .05 of the true sum?
   b. Does the series in part (a) converge conditionally or absolutely?Justify your answer.

8.
   a. Write the third Taylor polynomial for the function $f(x) = \frac{9}{x^2}$ at 1.
   b. For the function f in part (a), use Taylor's Inequality to bound the remainder $R_3(x)$ on $[\frac{1}{3}, 2]$.

9.
   a. Find the Maclaurin series x for $\frac{3x}{1-4x}$.
   b. Using the fact that power series can be differentiated term by term, find a Maclaurin series for $\frac{3x}{(1-4x)^2}$.

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