

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

**Final Examination
2 ½ Hours**

Mathematics 143

Fall 2007

Instructions:

Answer all questions.

Show all work

1) Perform the indicated integrations:

(a) $\int \frac{5}{(x+1)(x^2+4)} dx$

(b) $\int \frac{\sqrt{4-x^2}}{x^2} dx$

(c) $\int \sin^{-1} 3x \, dx$

(d) $\int \sqrt{\cos x} \sin^3 x \, dx$

2) Determine the following limits:

(a) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x^2 - \sin x}$

(b) $\lim_{x \rightarrow +\infty} (2e^x + x^2)^{\frac{3}{x}}$

3) (a) Evaluate $\int_0^{+\infty} x e^{-x} dx$ or show that it diverges.

(b) (i) Does the sequence $\left\{ \frac{2n}{n+1} \right\}_{n=1}^{\infty}$ converge or diverge? Justify your answer.

(ii) Does the series $\sum_{n=1}^{\infty} \frac{2n}{n+1}$ converge or diverge? Justify your answer.

4) In this question, give your answers in (a) and (b) correct to 4 decimal places.

(a) Use your calculator to find the length of the curve $y = \cos x$, where $0 \leq x \leq \frac{\pi}{4}$.

(b) (i) Show that the series $\sum_{n=5}^{\infty} \frac{2^n - n^2}{5^n + n}$ converges. Justify your answer.

(ii) Use your calculator to find an approximation for the sum of the series in (i).

5) Determine if each of the following series converges or diverges. If the series converges, determine its sum.

(a) $\frac{1}{e^2} + \frac{1}{e^4} + \frac{1}{e^6} + \dots$

(b) $1 - \frac{e^2}{2!} + \frac{e^4}{4!} - \frac{e^6}{6!} + \dots$

(c) $\frac{3}{2} + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \dots$

(continued on other side)

6) Test each series for convergence or divergence. **Justify your answers.**

(a) $\sum_{k=1}^{+\infty} \frac{(k!)^2}{(2k)!}$

(b) $\sum_{k=1}^{+\infty} \frac{\sin k}{k^3}$

(c) $\sum_{k=1}^{+\infty} \frac{1}{\sqrt{k}+1}$

(d) $\frac{1}{3} - \frac{2}{4} + \frac{3}{5} - \frac{4}{6} + \dots$

(e) $\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$

7) Determine the interval of convergence of the power series

$$\sum_{k=1}^{+\infty} \frac{k}{k^2+1} (x-2)^k$$

8) (a) Write a Maclaurin series for $f(x) = \cos 2x$

(b) Differentiate your answer in (a) to form an infinite series for $\sin 2x$.

(c) Use the first two nonzero terms of your series in (b) to approximate $\sin(2^\circ)$.

Hint: Remember to use radian measure.

(d) Estimate the largest possible error that you can make by using the approximation in (c). Justify your answer by citing an appropriate theorem.