QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 143 **Spring 2007**

Instructions: **Answer all questions.** **Show all work.**

1. Evaluate:

a) $\lim_{x\to 0^+} \dfrac{1-\ln x}{e^{\frac{1}{x}}}$

b) $\lim_{x\to 0^+} x^{\frac{1}{\ln x}}$

2. Evaluate:

a) $\int (\ln x)^2\, dx$

b) $\int \sec^5 x \sin x\, dx$

c) $\int_0^{\frac{2\sqrt{2}}{3}} \dfrac{dx}{(16-9x^2)^{3/2}}$

d) $\int \dfrac{2x^5-x^3-1}{x^3-x^2}\, dx$

e) $\int \dfrac{dx}{2+2\sqrt{x}}$

3. Evaluate:

a) $\int_{-\infty}^{\infty} \dfrac{x}{(x^2+3)^2}\, dx$

b) $\int_0^1 \dfrac{dx}{\sqrt{1-x^2}}$

4. Determine whether each of the following sequences is convergent or divergent. If it converges, find its limit.

a) $\left\{(-1)^{n+1}\dfrac{2n}{3n+1}\right\}_{n=1}^{\infty}$

b) $\left\{\dfrac{n}{e^n}\right\}_{n=1}^{\infty}$

5. Find the sum: $\sum_{n=3}^{\infty} \dfrac{2^{n+3}}{4^{n-2}}$

6. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. State reasons for your answers.

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{n^n}{n!}$

b) $\sum_{n=1}^{\infty} \dfrac{\sin n}{n^3}$

c) $\sum_{n=3}^{\infty} (-1)^n \dfrac{\ln n}{n}$

d) $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n(n+1)}}$

(over)

7. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{4^n x^n}{n^2}$$

8. **a)** Find the first four non zero terms of the Maclaurin series of $f(x)=\sin 2x$.
   b) Use the result of part a) to approximate $\sin(\pi/5)$. (Hint: choose $x=\pi/10$).