QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 151 **Fall 2006**

Instructions: **Answer all questions.** **Show all work.**

1) Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$ or $-\infty$ or does not exist, explain why.

(a) $\displaystyle\lim_{x\to -5^+} \frac{x^2-2x-3}{x^2+2x-15}$.

(b) $\displaystyle\lim_{x\to 1} \frac{\sqrt{2x+1}-\sqrt{3}}{x-1}$.

(c) $\displaystyle\lim_{x\to +\infty} \frac{1+8x-7x^2}{4x^2-3}$.

2) Use your calculator to construct a table to find $\displaystyle\lim_{x\to 0^+}(1+2x)^{1/x}$, correct to three decimal places. Include at least five appropriately chosen x – values to justify your answer and copy the resulting table into your booklet.

3) Use the definition of the derivative to find $f'(x)$ when $f(x)=\dfrac{2}{x-3}$.

4) Let

$$g(x)=\begin{cases} \dfrac{a}{x+5} & \text{if } 0\le x\le 1 \\ 3 & \text{if } 1<x\le 4 \\ x^2+bx-33 & \text{if } x>4 \end{cases}$$

Find values of "a" and "b" such that g is continuous for all $x\ge 0$.

5) Find $\frac{dy}{dx}$ for each of the following. In parts (a), (b) & (c), algebraic simplification is not necessary.

(a) $y=\left(8\sqrt[3]{x} - x^{-\frac{3}{2}}\right)^5$

(b) $y=\sqrt[4]{\dfrac{\sin x}{1+\cos x}}$

(c) $y=\tan^4\left(2x^6\right)$

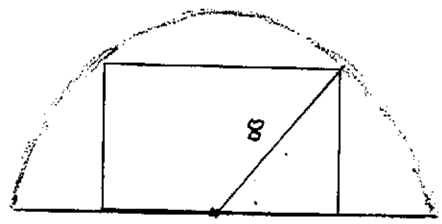
(d) $y=\displaystyle\int_4^{5x^2}\sqrt{t^3+\pi}\,dt$

6) Find an equation of the line tangent to $x^3y^5+2y^6+15x^3=16$ at the point $(1,-1)$.

7) At 6 P.M., a hot air balloon is 32 yards directly above an ant. If the balloon is descending vertically at the rate of 40 yards per hour and the ant is crawling east at 10 yards per hour, how fast will the distance between the ant and the balloon be changing at 6:30 P.M.? Interpret your answer.

(over)

8) Using calculus, find the maximum area that a rectangle can have if it is inscribed in a semicircle of radius 8, assuming the base of the rectangle lies along the diameter of the circle. (See picture below.)



9) Use the methods of calculus to sketch the graph of $y=f(x)=\frac{x+4}{x^2}$. Find intervals on which f is increasing, decreasing, concave upward, and concave downward. Be sure to identify all asymptotes, relative extrema and points of inflection.

10) State the Intermediate Value Theorem. Then use it to show that the function $f(x)=2\cos x-3x$ has a zero on the interval $[0,1]$. Use your graphing calculator to find this zero, accurate to five decimal places.

11) Let R be the region bounded by the graphs of $y=f(x)=x^2$, $x=1$, and the $x-$axis. Compute the area of R

(a) by setting up the limit of a Riemann sum. (Note: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$)

(b) by using a definite integral.

12) Integrate:

(a) $\int \frac{x^8-\sqrt{x}+1}{x^3}\,dx$

(b) $\int \frac{x^2+\cos 3x}{\sqrt{x^3+\sin 3x}}\,dx$