Department of Mathematics, Queens College
Math 151 Final Exam, Fall 2007

This exam has two parts. You have $2\frac{1}{2}$ hours to answer the questions. You must show all your work in the provided exam book.

PART I: CLEARLY WRITE THE LETTER OF THE CORRECT ANSWER IN YOUR EXAM BOOK. EACH QUESTION HAS 3 POINTS.

1. $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x^2-3x-4}$

(A) is $\dfrac{1}{20}$ (B) is $\dfrac{1}{4}$ (C) is 0 (D) does not exist

2. $\lim_{x\to 1^-} \dfrac{|x^2-3|}{x-1}$

(A) is 0 (B) is ∞ (C) is $-\infty$ (D) does not exist

3. If the function $f(x) = \begin{cases} \dfrac{\sin^2(5x)}{x^2} & \text{if } x \neq 0 \\ a & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then a is

(A) 5 (B) $\dfrac{1}{5}$ (C) 25 (D) $\dfrac{1}{25}$

4. Suppose f is a continuous function such that $f(0) = 2$, $f(2) = -1$ and $f(10) = 1$. Then, in the interval $[0, 10]$, the equation $f(x) = 0$ has

(A) at most two solutions
(B) at least two solutions
(C) exactly two solutions
(D) no solution at all

5. $\lim_{h\to 0} \dfrac{\sqrt[3]{27+h}-3}{h}$ describes the derivative $f'(a)$, where

(A) $f(x) = \sqrt[3]{27+x}$ and $a = 3$
(B) $f(x) = \sqrt[3]{27+x}$ and $a = 27$
(C) $f(x) = \sqrt[3]{x}$ and $a = 3$
(D) $f(x) = \sqrt[3]{x}$ and $a = 27$

6. If ε is very small, the best linear approximation to $\cos\left(\frac{\pi}{3}+\varepsilon\right)$ is

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\varepsilon$ (C) $\dfrac{1}{2} - \sqrt{3}\,\varepsilon$ (D) $\dfrac{1}{2} - 2\sqrt{3}\,\varepsilon$

7. My calculator suggests that the global maximum of the function $f(x) = \dfrac{\sin x}{x^2+1}$ on $(-\infty, \infty)$ is about

(A) 0.431 (B) 0.433 (C) 0.435 (D) 0.437

8. The position of an object moving along a line is given by $s(t) = \int_0^t \tan(x)\,dx$, where s is measured in feet and t is the time measured in seconds. The acceleration of the object (in ft/sec^2) at $t = \frac{\pi}{4}$ is

(A) 0 (B) 1 (C) -1 (D) 2

continued on the other side $\longrightarrow$

9. If $\int_a^b f(x)\,dx = 1$ and $\int_c^b f(x)\,dx = -2$, then $\int_c^a f(x)\,dx$ is

(A) -3 (B) -1 (C) 1 (D) 3

10. The area of the region in the plane enclosed by the curve $y = \sqrt{x}$, the x-axis, and the lines $x = 1$ and $x = 16$ is

(A) 12 (B) 21 (C) 42 (D) 63

PART II: SOLVE THE FOLLOWING 5 PROBLEMS. SHOW ALL YOUR WORK.

Problem 1. [20 points] In each case, find the derivative $y' = \dfrac{dy}{dx}$ (simplifying your answer is optional):

(i) $y = \sqrt{x + \sqrt{x}}$ (ii) $y = \left(\dfrac{x}{1+\sin x}\right)^3$

(iii) $y = (x^3+1)^5 \cos(x^2)$ (iv) $x^2y^2 - \sin y + 6x = 0$

Problem 2. [10 points] The base radius of a cylinder is expanding at the rate of 1 in/min while its height is shrinking at the same rate. How fast is the volume of this cylinder changing at the moment when the base radius is 10 in and the height is 4 in? Is the volume increasing or decreasing at that moment? (The volume V of a cylinder of base radius r and height h is given by the formula $V = \pi r^2 h$.)

Problem 3. [15 points] Consider the function $f(x) = \dfrac{x-2}{x^3}$.

(i) Find the domain of f and the vertical and horizontal asymptotes of its graph.

(ii) Find the formulas for f' and f'' and use them to determine the intervals of increase/decrease and concavity of f. Make sure you clearly identify the critical point(s) and their type (i.e., local max, local min, neither) as well as the inflection point(s).

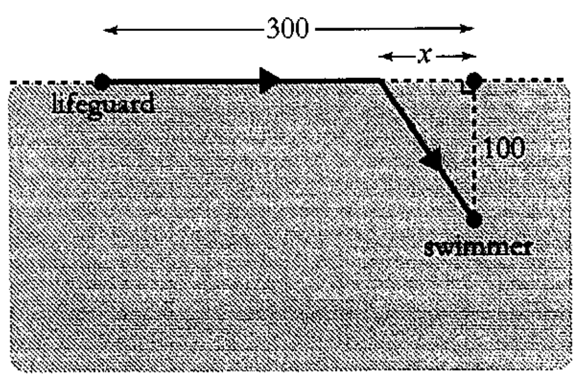
(iii) Use your calculator to graph f in an appropriate window and copy what you see in your exam book. Explain why the features of this graph are consistent with your findings in (i) and (ii).

Problem 4. [15 points] A swimmer in need of help is in the ocean 100 feet from the shoreline. A lifeguard is on the shoreline 300 feet from the point on the shoreline closest to the swimmer. He decides to reach the swimmer by first running along the shoreline and then swimming directly toward him. The lifeguard can run at 10 ft/sec and swim at 6 ft/sec.

(i) Express the total time T it takes for the lifeguard to reach the swimmer as a function of the length x shown in the picture.

(ii) Using calculus, find the value of x which minimizes $T(x)$ on the interval $0 \le x \le 300$.

(iii) What is the quickest the lifeguard can reach the swimmer?



Problem 5. [10 points] Evaluate the following indefinite integrals:

(i) $\displaystyle\int \frac{x^2+2}{\sqrt{x^3+6x}}\,dx$ (ii) $\displaystyle\int (1-\sin(2x))^4 \cos(2x)\,dx$