

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 151

Spring 2007

Instructions: Answer all questions. Show all work.

1. Find each limit. If a limit does not exist, briefly explain why.

a) $\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x^2 - 9}$

b) $\lim_{x \rightarrow 4} \frac{\frac{1}{x} - \frac{1}{4}}{x - 4}$

c) $\lim_{x \rightarrow \infty} \frac{2x - \sin(x) + 1}{\cos(x) + 3x - 2}$

d) $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{\tan(x)}$

2. Use your calculator to approximate $\lim_{x \rightarrow 0} (1-x)^{\left(\frac{3}{x}\right)}$ correct to 5 decimal places. (Use the TABLE function and display enough values of x and $y(x)$ to justify your answer to 5 decimal places.)

3. Let $f(x) = \frac{1}{3x-2}$. Find $f'(x)$ using the definition of the derivative.

4. Find the derivatives:

a) $y = 7x^4 - 3x^2 + 4x - \pi^2$

b) $y = (x^4 - 2x^3 + 5)^3 (x^3 - 4x + 3)^4$

c) $y = \sin\left(\frac{x^2 - 2}{x^2 + 1}\right)$

d) $x^3 + y^3 = 2xy$

e) $f(x) = \int_1^{x^2} \cos(t^4 + 1) dt$

5. Let $f(x) = \frac{x^2 - 1}{x^2 - 4}$.

- a) Find $f'(x)$ and $f''(x)$ and simplify.
- b) Find any critical points and any horizontal or vertical asymptotes.
- c) Find the intervals where f is increasing/decreasing.
- d) Find the intervals where f is concave up/down and any inflection points.
- e) Sketch the graph using this information. (Your sketch should agree with your answers in parts a) to d).

6. Let $f(x) = x^5 + 5x^3 + x + 1$.

- a) Find $f'(x)$, any critical points of f and the values of f at $x = -1$ and at $x = 0$.
- b) Use your answers in a) to show away that f must have exactly one real root.
- c) Use your calculator to find this root correct to 3 decimal places.

7. A 13-foot ladder leans against a vertical wall so the ladder, ground, and wall form a right triangle. The bottom of the ladder starts to slide away from the wall at a constant rate of 6 feet per second.

- a) How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 feet from the wall?
- b) At this moment what is the rate of change of the area of the triangle formed by the wall, ladder and ground?

(continued on other side)

8. Find the minimum possible perimeter for a rectangle whose area is 16 square inches, and find its dimensions. (The perimeter of a rectangle is the sum of the lengths of all four sides).
9. a) Find an equation of the tangent line to the curve $y = \sqrt{x}$ at the point where $x = 9$. Sketch the curve and its tangent line at this point.
 b) Use the equation of this tangent line (linearization) to approximate $\sqrt{8.98}$. Is this an overestimate or an underestimate?
10. Find the integrals:

a) $\int x^5 (x^2 + 1) dx$

b) $\int_0^{\frac{\pi}{2}} \cos(\sin(x)) \cos(x) dx$

11. Find the integral as a limit of Riemann sums:

$$\int_0^2 (2x+1) dx$$

(Hint: Use equal subintervals and right hand endpoint sample points.)

$$\left(\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad ; \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \right)$$

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