QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 152 **Fall 2006**

Instructions: **Answer all questions. Show all work**

1) Let R be the region in the plane bounded by the graphs of $y=e^x$, $y=e^{-x}$, and $x=\ln 2$.

(a) Set up a definite integral that computes the area of R. WITHOUT USING YOUR CALCULATOR, find this area.
(b) Set up a definite integral that computes the volume of the solid of revolution obtained when R is rotated about the $y-axis$. You need not evaluate the integral.
(c) Set up a definite integral that computes the volume of the solid of revolution obtained when R is rotated about the line $y=-1$. You need not evaluate the integral.

2) Let $f(x)=\frac{1}{2}x^2-\frac{1}{4}\ln x$, where $x>\frac{1}{2}$.

(a) Use a derivative to show that f is one-to-one and thus has an inverse function. Then compute $\left(f^{-1}\right)'\left(\frac{1}{2}\right)$. [Note: You need not find f^{-1} explicitly.]
(b) Find the exact value of the length of the arc of the graph of $y=f(x)$ for $2\le x\le 4$.

3) If $f(x)=\left(x^2\right)^{\left(e^{-x}\right)}$, find

(a) $f'(x)$ (b) $\lim\limits_{x\to+\infty} f(x)$

4) Integrate:

(a) $\int e^{-3x}\cos x\,dx$

(b) $\int \frac{\tan^3\left(\frac{1}{x}\right)\sec^3\left(\frac{1}{x}\right)}{x^2}\,dx$

(c) $\int \frac{x^2-x-1}{x\left(x^2+1\right)}\,dx$

(d) $\int \frac{dx}{\left(4-x^2\right)^{5/2}}$

5) (a) Evaluate $\int_{-1}^{0} \frac{e^{1/x^3}}{x^4}\,dx$, if it converges.

(b) The number of bacteria in a culture grows exponentially and thus obeys the law of exponential growth. If the original population triples in 5 days, how long did it take for the original population to double? Round your answer to the nearest hundredth.

6) (a) Show that the sequence $\left\{\frac{(2n)!}{(n!)^2 4^n}\right\}$ is decreasing.

(b) Find the exact value of each of the following series:

(i) $\frac{1}{1^2+1}+\frac{1}{2^2+2}+\frac{1}{3^2+3}+\ldots$

(ii) $\sum\limits_{k=0}^{+\infty} \frac{(-1)^k}{(2k)!}\left(\frac{\pi}{3}\right)^{2k}$

(over)

7) Determine the convergence or divergence of each of the following series. Give reasons for your conclusions

(a) $\sum_{k=1}^{+\infty} \frac{k!(k+2)!}{(2k+2)!}$

(b) $\sum_{k=2}^{+\infty} \frac{(-1)^k}{k(\ln k)^{2/3}}$

(c) $\sum_{k=1}^{+\infty} \tan^{-1}\left(\frac{k+1}{k+2}\right)$

(d) $\sum_{k=1}^{+\infty} \left(\frac{\sin k}{k}\right)^3$

8) Determine the interval of convergence of the power series $\sum_{k=0}^{+\infty} \frac{(2x-1)^k}{(k+3)7^k}$. Classify any convergence as either absolute or conditional.

9) Let $f(x)=x^{3/2}$.

(a) Find $T_3(x)$, the third Taylor polynomial of f at $x=4$.

(b) Use Taylor's inequality to bound $R_3(x)$, the third remainder of f. Then use your answer to determine the largest possible error that can result when $T_3(4.1)$ is used to approximate $f(4.1)$.

10) (a) Beginning with the Maclaurin series for $\frac{1}{1-x}$, write the Maclaurin series for $\frac{x}{1+x^6}$.

(b) Use the result of part (a) to obtain a series representation for $\int_0^{1/3} \frac{x}{1+x^6}\,dx$.

(c) Use the fewest possible number of terms of the series in part (b) to estimate the value of the definite integral with four-decimal-place accuracy.

(d) USE YOUR CALCULATOR to find the value of the definite integral in part (b).