QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
Final Examination
2 ½ Hours

Mathematics 152 **Fall 2007**

Instructions: **Answer all questions.** **Show all work.**

1) Let R be the region bounded by the curves $y = \tan\left(\frac{2x}{x^2+1}\right)$ and $y = .05x^2$.

a) Set up the integral for the volume of the solid obtained by rotating R around the line $y = -3$. Use your calculator to find the lower and the upper limits of the integration.

b) Evaluate the integral in b) using your calculator.

c) Use the method of cylindrical shells to set up the integral for the volume of the solid obtained by rotating R around the line $x = -1$.

d) Evaluate the integral in c) using the calculator.

2) Differentiate the following functions:

a) $f(x) = e^{(x^x)}$ **b)** $g(x) = \tan^{-1}\left(\frac{1}{1+x^2}\right)$

3) Find the following limits:

a) $\lim_{x \to 0^+} (\ln(1+x))^x$ **b)** $\lim_{x \to 0^-} x^2 e^{\frac{1}{x}}$ **c)** $\lim_{x \to 0^+} x^2 e^{\frac{1}{x}}$

4) Integrate:

a) $\int \frac{dx}{\sqrt{2x^2 - x}}$ **b)** $\int_0^e \sqrt{x} \ln \frac{1}{\sqrt{x}}\, dx$ **c)** $\int \frac{x(x+1)}{(x+2)^3}\, dx$

5) Find the arc length of the curve $f(x) = \frac{1}{3} \ln \cos 3x$ from $x = -\frac{\pi}{12}$ to $x = \frac{\pi}{9}$.

6) Assume that a bank compounds interest continuously. If an initial investment of $1,000 has value $1,200 in 18 months, how long does it take to triple the initial investment?

7) Find the limit of each of the following sequences or show that the sequence diverges.

a) $\left\{\frac{e^{-n^2}}{n}\right\}$ **b)** $\left\{\frac{\sqrt[3]{n}}{\sqrt[4]{n+1}}\right\}$

8) Determine the convergence or divergence of the following series. State the test that you are using.

a) $\sum_{n=1}^{\infty} \frac{1}{4n^2+8n-5}$ **b)** $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^2+n}}$

(continued on other side)

9) Find the radius and the interval of convergence for

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x-5)^n}{n+1}.$$

10) a) Starting with the Maclaurin series for $\sin x$, write the Maclaurin series for $\sin\left(x^2\right)$.

b) Find a series representation for $\int_0^2 \sin\left(x^2\right)dx$.

c) Using the series found in part a), evaluate the integral in b) so that the error is less than .0001.

d) Using the series found in part a), compute $\lim_{x\to 0}\frac{\sin x^2 - x^2}{x^6}$.