QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 152 **Spring 2007**

Instructions: **Answer all questions.** **Show all work.**

1. Let R be the region in the plane bounded by the curve $y=\sqrt{x+2}$ and the lines $y=x$ and $y=0$.

   a) Sketch region R and determine its area.
   b) Set up, but do not evaluate, the definite integral representing each of the following:
      (*i*) The volume generated by rotating R about the x-axis using the method of cylindrical shells.

      (*ii*) The volume generated by rotating R about the line $x=3$ using the "disk-washer" method.

2. Find y'.

   a) $y=3^x+e^{x^2}$

   b) $y=\left(\sin^{-1}x\right)^x$

3. Evaluate each of the following integrals. If the improper integral diverges, state this as your answer.

   a) $\int(\ln x)^2\,dx$

   b) $\int\sqrt{\tan x}\,\sec^4 x\,dx$

   c) $\int_e^{+\infty}\dfrac{dx}{x(\ln x)^3}$

   d) $\int e^x\sqrt{1-e^{2x}}\,dx$

   e) $\int\dfrac{4x^2+2x+3}{x\left(x^2+1\right)}$

4. Let $f(x)=4x+\cos x-\sin x$.

   a) Use a derivative to show that $f(x)$ is one-to-one and thus has an inverse function.

   b) Given that $\left(\dfrac{\pi}{4},\,\pi\right)$ is a point on the graph of f, compute $g'(\pi)$ where g is is the inverse of f.

5. a) Find the length of the curve of $y=\dfrac{1}{3}\left(x^2+2\right)^{3/2}$ from $x=0$ to $x=3$.

   b) Find $\lim\limits_{x\to 0^+}\left(\cos\sqrt{x}\right)^{\frac{1}{x}}$.

(over)

6. The half-life of radioactive carbon-14 is 5730 years. If 100 grams of radioactive carbon-14 are stored in a cave, how many grams will be left after 1000 years? (Round answer to nearest tenth.)

7. Find the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{(x-3)^n}{(n+4)5^n}$.

8. Suppose $f(x) = x \ln x$.

   a) Find $T_4(x)$, the fourth Taylor polynomial, centered at $a=1$.

   b) Use Taylor's Inequality to estimate the largest possible error that can result in approximating $f(x)$ by $T_4(x)$ when x is in the interval $.8 \le x \le 1.2$.

   c) Use $T_4(x)$ (and your calculator) to approximate $1.2\,(\ln 1.2)$ to the nearest thousandth.

9. a) Beginning with the Maclaurin series for e^x, write the Maclaurin series for e^{-x^3}.

   b) Find a series representation for

   (*i*) $\int e^{-x^3}\,dx$

   (*ii*) $\int_0^1 e^{-x^3}\,dx$

   c) Use the fewest number of terms of the series for $\int_0^1 e^{-x^3}\,dx$ to evaluate the definite integral so that the error will be less than .001 .

   d) Use your calculator to find the value of $\int_0^1 e^{-x^3}\,dx$ to 4 decimal places.

10. Determine the convergence or divergence of the following series. In the case of an alternating series, determine if the convergence is absolute or conditional. In each case, state what test was used.

   a) $\sum_{k=1}^{\infty} k^2\, e^{-k^3}$

   b) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

   c) $\sum_{n=1}^{\infty} \frac{3^n n!}{2^n\, n^n}$

   d) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+2}{k(k+1)}$.