## Final Examination

Please show all your work and write so that I can read. Justify your answers. Do all work on the test paper. Write your name on top of each page in the space provided. Good luck!

Please print your name here: $\qquad$
(1) Determine the domain of the function

$$
f(x, y)=\sqrt{x^{2}+y^{2}-1}+\ln \left(4-x^{2}-y^{2}\right)
$$

(2) Show that the function $f(x, y)=\ln \sqrt{x^{2}+y^{2}}$ satisfies $f_{x x}(x, y)+f_{y y}(x, y) \equiv 0$.
(3) Compute an approximate value of $\sqrt{20-(1.95)^{2}-7(1.08)^{2}}$ using linear approximation at the point $\left(x_{0}, y_{0}\right)=(2,1)$.
$\qquad$
(4) Find the critical points of the function $f(x, y)=x^{4}+y^{4}-4 x y+1$. Determine their character (local minimum, local maximumu, saddle point) if possible.
(5) Find the points on the surface of the ellipsoid $x^{2}+y^{2}+2 z^{2}=1$ where the normal vector is parallel to the vector from $(0,1,1)$ and $(-2,3,5)$.
(6) Evaluate the integral

$$
\iiint_{E} x e^{\left(x^{2}+y^{2}+z^{2}\right)^{2}} d V
$$

where $E$ is the solid between the spheres $x^{2}+y^{2}+z^{2}=1$ and $x^{2}+y^{2}+z^{2}=4$ in the first quadrant.

Your name: $\qquad$
(7) Evaluate the integral

$$
\int_{0}^{1} \int_{3 y}^{3} e^{x^{2}} d x d y
$$

by reversing the order of integration.
(8) Find the absolute maximum and minimum values of the function $f(x, y)=x^{4}+y^{4}-4 x y+2$ on the set $D=\{(x, y) \mid 0 \leq x \leq 3,0 \leq y \leq 2\}$.
(9) The acceleration of a moving object is given by $\vec{a}(t)=\left\langle\cos (2 t), \sin (t), e^{t}\right\rangle$. Compute its velocity $\vec{v}(t)$ and position $\vec{r}(t)$ if the initial velocity is $\vec{v}(0)=\langle 1,0,0\rangle$ and the initial position is $\vec{r}(0)=\langle 0,1,1\rangle$. (Please use arrows over vectors to avoid confusion between vectors and scalars!)
(10) Find an equation of the plane passing through three points $(-1,2,0),(3,-2,-1)$ and $(3,-5,1)$.

