Mathematics 231 Prof. J. Dodziuk October 6, 2016

Test #1

Please show all your work and write so that I can read. Justify your answers. Do all work on the test paper. Write your name on top of each page in the space provided. Good luck!

Please *print* your name here: ________________________________

1. Suppose that an $n \times n$ matrix A satisfies the equation

$$A^5 - 5A^4 + 4A^3 - A^2 + A + I = 0.$$

Show that A is invertibble and give a formula for A^{-1} in terms of A.

2. Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}.$$

3. Find the values of c, if any, for which the matrix below is invertible.

$$\begin{bmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 0 & 1 & c \end{bmatrix}$$

Your name: ______________________________

4. Verify that

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & -4 \end{bmatrix}^{-1} = \frac{1}{9}\begin{bmatrix} -5 & -1 & -1 \\ 1 & -7 & 2 \\ -1 & -2 & -2 \end{bmatrix}$$

and solve the matrix equation

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & -4 \end{bmatrix} X = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 6 & 7 & 8 & 9 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

for X.

5. Let $A\mathbf{x} = 0$ be a system of n linear equations with n unknowns, and let Q be an invertible $n \times n$ matrix. Prove that $A\mathbf{x} = 0$ has only the trivial solution if and only if $(QA)\mathbf{x} = 0$ has only the trivial solution.

6. Determine the conditions on the b_i's, if any, in order to guarantee that the system below is consistent.

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & - & x_3 & = & b_1 \\ -4x_1 & + & 5x_2 & + & 3x_3 & = & b_2 \\ -2x_1 & + & x_2 & + & x_3 & = & b_3 \end{array}$$

Your name: ____________________

7. Show that if A is a symmetric $n \times n$ matrix and B is a $k \times n$ matrix then the the matrix BAB^T is a $k \times k$ symmetric matrix.

8. Find the matrix of the linear transformation $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^2$ defined by

$$\begin{array}{rcrcrcrcr} w_1 & = & 2x_1 & + & 3x_2 & - & 5x_3 & - & x_4 \\ w_2 & = & x_1 & - & 5x_2 & + & 2x_3 & - & 3x_4. \end{array}$$

and then compute $T(1, -1, 2, 4)$ using matrix multiplication.

9. Let $T_A : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be multiplication by the matrix

$$A = \begin{bmatrix} -1 & 3 & 0 \\ 2 & -1 & 5 \\ 7 & 1 & -1 \end{bmatrix}$$

and let $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ be the standard basis vectors for $\mathbb{R}^3$. Find $T_A(\mathbf{e}_1 + \mathbf{e}_3)$.

10. Find the quadratic polynomial whose graph passes through the points $(0, 0)$, $(-1, 1)$, and $(1, 2)$.