1. Which of the following is not a field? Explain.
   (a) The integers $\mathbb{Z}$
   (b) The rational numbers $\mathbb{Q}$
   (c) The real numbers $\mathbb{R}$
   (d) The complex numbers $\mathbb{C}$

2. Which of the following is not a field? Explain.
   (a) The numbers $\{0, 1\}$ with $+$ and $\times$ defined “mod 2”.
   (b) The numbers $\{0, 1, 2\}$ with $+$ and $\times$ defined “mod 3”.
   (c) The numbers $\{0, 1, 2, 3\}$ with $+$ and $\times$ defined “mod 4”.
   (d) The numbers $\{0, 1, 2, 3, 4\}$ with $+$ and $\times$ defined “mod 5”.

3. Let $\alpha \in \mathbb{C}$ be nonzero. Define the number $\frac{1}{\alpha}$ and prove that $\frac{1}{\alpha} = \alpha$.

4. Express $\frac{1}{4+5i}$ in the form $a+bi$ for real numbers $a, b$.

5. True or False:
   (a) There is only one number $\alpha \in \mathbb{R}$ so that $\alpha^3 = 2$.
   (b) There is only one number $\alpha \in \mathbb{C}$ so that $\alpha^3 = 2$.

6. True or False:
   (a) There exists a number $\alpha \in \mathbb{R}$ so that $\alpha^2 = -2$.
   (b) There exists a number $\alpha \in \mathbb{C}$ so that $\alpha^2 = -2$.

7. Does there exist a number $\alpha \in \mathbb{C}$ so that $\alpha(1 + I, 2, 2 + 2I, 3-2I) = (2, 2 - 2I, 4, 1-5I)$?

8. Let $V$ be a vector space over a field $F$. Prove that
   (a) For all $v \in V$, $0v = 0$.

      Note: the zero on the left is the zero scalar in $F$ and the zero on the right is the zero vector in $V$.

   (b) For all $v \in V$, $(-1)v = -v$.

      Note: the $-1$ on the left is a scalar in the field $F$, the $-v$ on the right is the additive inverse of the vector $v \in V$.
9. Using the correspondence \( a + bi \longleftrightarrow (a, b) \) complex numbers can be identified with points in the Cartesian plane. The four points pictured below correspond to \( z, w, z + w, \) and \( zw \) for two complex numbers \( z, w \in \mathbb{C} \). Which are which?

![Diagram showing four points on a Cartesian plane, labeled \( z, w, z + w, \) and \( zw \).]

10. Consider the vector space \( \mathbb{R}^4 \). Which of the following subsets are subspaces?

   (a) \( \{(a, b, c, d) \in \mathbb{R}^4 : a + b + c = 0\} \)
   
   (b) \( \{(a, b, c, d) \in \mathbb{R}^4 : abc = 0\} \)
   
   (c) \( \{(a, b, c, d) \in \mathbb{R}^4 : a \geq 0\} \)
   
   (d) \( \{(a, b, c, d) \in \mathbb{R}^4 : a = 2\} \)
   
   (e) \( \{(a, b, c, d) \in \mathbb{R}^4 : a = d\} \)
   
   (f) \( \{(a, b, c, d) \in \mathbb{R}^4 : a + b + 1 = c\} \)
   
   (g) \( \{(a, b, c, d) \in \mathbb{R}^4 : a + b = 2c\} \)

11. Consider the vector space \( \mathbb{R}^R \). Which of the following subsets are subspaces?

   (a) \( \{f : \mathbb{R} \to \mathbb{R} : f(1) = 1\} \)
   
   (b) \( \{f : \mathbb{R} \to \mathbb{R} : f(1) = 0\} \)
   
   (c) \( \{f : \mathbb{R} \to \mathbb{R} : f \text{ is onto}\} \)
   
   (d) \( \{f : \mathbb{R} \to \mathbb{R} : f \text{ is continuous}\} \)
   
   (e) \( \{f : \mathbb{R} \to \mathbb{R} : f \text{ is differentiable}\} \)
   
   (f) \( \{f : \mathbb{R} \to \mathbb{R} : f''(x) = f(x)\} \)
12. Let $V = \mathbb{R}^3$. Consider the following three subspaces of $V$

$$W = \{(0,0,a) \in V : a \in \mathbb{R}\}$$
$$X = \{(a,a,a) \in V : a \in \mathbb{R}\}$$
$$Y = \{(a,b,c) \in V : a + b + c = 0\}$$
$$Z = \{(a,a,b) \in V : a, b \in \mathbb{R}\}$$

True or False:

(a) $(1,1,-2) \in W$
(b) $(1,1,-2) \in X$
(c) $(1,1,-2) \in Y$
(d) $(1,1,-2) \in Z$
(e) $W$ is a subspace of $X$
(f) $W$ is a subspace of $Y$
(g) $W$ is a subspace of $Z$
(h) $X$ is a subspace of $Z$
(i) $W$ is a subspace of $Z$
(j) $W \cap X = \{(0,0,0)\}$
(k) $X \cap Z = X$
(l) $Z = W + X$
(m) $Z = W \oplus X$
(n) $V = Y + Z$
(o) $V = Y \oplus Z$