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## Part I: True/False. 1 point each

1. If $x, y, z$ are elements of a group $G$ and $x y=x z$ then $y=z$.
2. If $f, g, h$ are functions from a set $X$ to itself and $f g=f h$ then $g=h$.
3. The function $f:(\mathbb{Z} / 7 \mathbb{Z})^{\times} \rightarrow(\mathbb{Z} / 11 \mathbb{Z})^{\times}$defined by $f(x)=5 x \bmod 11$ is a left-invertible function.
4. The function $f:(\mathbb{Z} / 7 \mathbb{Z})^{\times} \rightarrow(\mathbb{Z} / 11 \mathbb{Z})^{\times}$defined by $f(x)=5 x \bmod 11$ is a group homomorphism.
5. $10 x \equiv 1 \bmod 21$ has a solution $x \in \mathbb{Z} / 21 \mathbb{Z}$.
6. There exists a homomorphism $g: D_{8} \rightarrow S_{4}$ with $g\left(R_{90}\right)=(1234)$ and $g(H)=(12)$.
7. GL $(2,3)$ has order 48.
8. True or False: The function $\phi: \operatorname{GL}(2,7) \rightarrow(\mathbb{Z} / 7 \mathbb{Z})^{\times}$defined by $\phi(M)=\operatorname{det}(M)$ is a homomorphism.
9. For any group $G$ and any element $x \in Z(G)$ we have $C_{G}(x)=G$.
10. For any homomorphism $\phi: G \rightarrow H$, the set $K=\{g \in G: \phi(g)=e\}$ is a subgroup of $G$.
11. If $G$ is a group with the property that $(a b)^{2}=a^{2} b^{2}$ for any $a, b \in G$, then $G$ is abelian.
12. The permutation $(653124)(5421)$ has order two in $S_{6}$.
13. $\mathbb{Z} / 2 \mathbb{Z} \times \mathbb{Z} / 2 \mathbb{Z} \times \mathbb{Z} / 2 \mathbb{Z}$ has twelve subgroups.
14. The quaternions $Q$ and the dihedral group $D_{8}$ are isomorphic.
15. GL( 2,2 ) and $S_{3}$ are isomorphic.
16. $\mathbb{Z} / 6 \mathbb{Z}$ are and $\mathbb{Z} / 2 \mathbb{Z} \times \mathbb{Z} / 3 \mathbb{Z}$ are isomorphic.
17. $(Z / 7 \mathbb{Z})^{\times}$and $\mathbb{Z} / 6 \mathbb{Z}$ are isomorphic.

## Part II: Short Answer. 3 points

18. Choose one of the True / False problems and write a complete justification of your answer.
