18

## Name

## Part I: True/False. 1 point each

- **1.** If *x*, *y*, *z* are elements of a group *G* and xy = xz then y = z.
- **2.** If f, g, h are functions from a set X to itself and fg = fh then g = h.
- **3.** The function  $f : (\mathbb{Z}/7\mathbb{Z})^{\times} \to (\mathbb{Z}/11\mathbb{Z})^{\times}$  defined by  $f(x) = 5x \mod 11$  is a left-invertible function.
- **4.** The function  $f : (\mathbb{Z}/7\mathbb{Z})^{\times} \to (\mathbb{Z}/11\mathbb{Z})^{\times}$  defined by  $f(x) = 5x \mod 11$  is a group homomorphism.
- **5.**  $10x \equiv 1 \mod 21$  has a solution  $x \in \mathbb{Z}/21\mathbb{Z}$ .
- 6. There exists a homomorphism  $g: D_8 \rightarrow S_4$  with  $g(R_{90}) = (1234)$  and g(H) = (12).
- **7.** GL(2, 3) has order 48.
- **8.** True or False: The function  $\phi : \operatorname{GL}(2,7) \to (\mathbb{Z}/7\mathbb{Z})^{\times}$  defined by  $\phi(M) = \det(M)$  is a homomorphism.
- **9.** For any group *G* and any element  $x \in Z(G)$  we have  $C_G(x) = G$ .
- **10.** For any homomorphism  $\phi : G \to H$ , the set  $K = \{g \in G : \phi(g) = e\}$  is a subgroup of G.
- **11.** If *G* is a group with the property that  $(ab)^2 = a^2b^2$  for any  $a, b \in G$ , then *G* is abelian.
- **12.** The permutation (653124)(5421) has order two in  $S_6$ .
- **13.**  $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$  has twelve subgroups.
- 14. The quaternions Q and the dihedral group  $D_8$  are isomorphic.
- **15.** GL(2, 2) and  $S_3$  are isomorphic.
- **16.**  $\mathbb{Z}/6\mathbb{Z}$  are and  $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$  are isomorphic.
- 17.  $(Z/7\mathbb{Z})^{\times}$  and  $\mathbb{Z}/6\mathbb{Z}$  are isomorphic.

## Part II: Short Answer. 3 points

18. Choose one of the True / False problems and write a complete justification of your answer.