## MIDTERM EXAM ANSWERS

## **MATH 333**

**1.** Which of the following diagrams says "g is a right inverse of f"?

$A \xrightarrow{g} B$	$A \xrightarrow{f} B$	$B \xrightarrow{f} A$	$B \xrightarrow{\operatorname{id}_B} B$
$\operatorname{id}_A \downarrow f$	$\operatorname{id}_A$	$d_B$	f g
A	A	B	Α

**Answer.** Only the first diagram. The first diagram says  $fg = id_A$ , which means that g is a right inverse of f. The second says  $gf = id_A$ , the third says  $gf = id_B$ , and the fourth says  $gf = id_B$  all three of which say g is a left inverse of f.

**2.** If we let *e* be the identity permutation,  $f = (1 \ 2 \ 3)$  and  $g = (1 \ 2)$  then the group  $S_3$  can be presented as

$$S_3 = \langle f, g | f^3 = g^2 = e, gf = f^2 g \rangle.$$

By choosing the right matrices for e, f, and g, the group GL(2, 2) can also be presented as

$$GL(2,2) = \langle f, g | f^3 = g^2 = e, gf = f^2 g \rangle.$$

Choose the right matrices for e, f, and g:

Answer.

$$f = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \qquad e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad g = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The relations are straightforward to check.

- 3. Solve the equations, or explain why no solution exists:
  - (a) Solve 5x + 6 = 10 for  $x \in \mathbb{Z}/11\mathbb{Z}$

Answer. Here, x = 3 is a solution since  $(5)(3) + 6 = 21 \equiv 10 \mod 11$ . To find this solution, I added -6 to both sides to get 5x = 4. Then I multiplied both sides by 9 to get  $(9)(5)x = (9)(4) \mod 11 \Rightarrow x = 3 \mod 11$  since  $(9)(5) = 1 \mod 11$  and  $(9)(4) = 3 \mod 11$ .

(b) Solve 5x + 6 = 10 for  $x \in \mathbb{Z}/12\mathbb{Z}$ 

Answer. Here, x = 8 is a solution since  $(5)(8) + 6 = 46 \equiv 10 \mod 12$ . To find this solution, I added -6 to both sides to get 5x = 4. Then I multiplied both sides by 5 to get  $(5)(5)x = (5)(4) \mod 12 \Rightarrow x = 8 \mod 12$  since  $(5)(5) = 1 \mod 12$  and  $(5)(4) = 8 \mod 12$ .

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