## MIDTERM EXAM ANSWERS

## MATH 333

1. Which of the following diagrams says " $g$ is a right inverse of $f$ " ?


Answer. Only the first diagram. The first diagram says $f g=\mathrm{id}_{A}$, which means that $g$ is a right inverse of $f$. The second says $g f=\mathrm{id}_{A}$, the third says $g f=\mathrm{id}_{B}$, and the fourth says $g f=\mathrm{id}_{B}$ all three of which say $g$ is a left inverse of $f$.
2. If we let $e$ be the identity permutation, $f=\left(\begin{array}{ll}123)\end{array}\right.$ and $g=(12)$ then the group $S_{3}$ can be presented as

$$
S_{3}=\left\langle f, g \mid f^{3}=g^{2}=e, g f=f^{2} g\right\rangle .
$$

By choosing the right matrices for $e, f$, and $g$, the group $G L(2,2)$ can also be presented as

$$
G L(2,2)=\left\langle f, g \mid f^{3}=g^{2}=e, g f=f^{2} g\right\rangle
$$

Choose the right matrices for $e, f$, and $g$ :
Answer.

$$
f=\left(\begin{array}{ll}
1 & 1 \\
1 & 0
\end{array}\right) \quad e=\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) \quad g=\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right)
$$

The relations are straightforward to check.
3. Solve the equations, or explain why no solution exists:
(a) Solve $5 x+6=10$ for $x \in \mathbb{Z} / 11 \mathbb{Z}$

Answer. Here, $x=3$ is a solution since $(5)(3)+6=21 \equiv 10 \bmod 11$. To find this solution, I added -6 to both sides to get $5 x=4$. Then I multiplied both sides by 9 to get $(9)(5) x=(9)(4) \bmod 11 \Rightarrow x=3 \bmod 11 \operatorname{since}(9)(5)=1 \bmod 11$ and $(9)(4)=3 \bmod 11$.
(b) Solve $5 x+6=10$ for $x \in \mathbb{Z} / 12 \mathbb{Z}$

Answer. Here, $x=8$ is a solution since (5)(8) $+6=46 \equiv 10 \bmod 12$. To find this solution, I added -6 to both sides to get $5 x=4$. Then I multiplied both sides by 5 to get $(5)(5) x=(5)(4) \bmod 12 \Rightarrow x=8 \bmod 12$ since $(5)(5)=1 \bmod 12$ and $(5)(4)=8 \bmod 12$.

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[^0]:    Date: October 31, 2017.

