

1. Describe the elements and product in  $D_8$  and complete the multiplication table. Explain why no element appears more than once in any row or any column.
2. Let  $n$  be a natural number and let  $a \in \mathbb{Z}/n\mathbb{Z}$ . Prove that  $a$  has a multiplicative inverse if and only if  $\gcd(a, n) = 1$ .

3. On page 7, the group  $D_8$  is described with generators and relations as

$$D_8 = \langle a, b \mid a^4 = b^2 = e, ba = a^3b \rangle.$$

On page 17, the group  $S_3$  is described with generators and relations as

$$S_3 = \langle f, g \mid f^3 = g^2 = e, gf = f^2g \rangle.$$

Give a description of  $\text{GL}(2, 2)$  with generators and relations.

4. Solve for  $x \in S_7$ :

$$(1, 2)(4, 5, 2)x = (7, 1, 3, 2, 5)$$

5. In class, we proved that the Well Ordering Axiom (WOA) of the natural numbers implies the Principle of Mathematical Induction (PMI). Prove the converse. That is, prove that the PMI proves the WOA.

6. Let  $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \in \text{GL}(2, 3)$  and  $b = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \in (\mathbb{Z}/3\mathbb{Z})^2$ . Solve for  $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in (\mathbb{Z}/3\mathbb{Z})^2$  in the equation  $Ax = b$ .

7. Let  $f : X \rightarrow Y$  be a function. Define what it means for  $f$  to be *right invertible*, *left invertible*, and *invertible*. Give examples of a function that is right invertible and not left invertible and function that is left invertible and not right invertible. Give an example of a function that is invertible and a function that is not invertible.

8. Prove that if  $f : X \rightarrow Y$  is both left invertible and right invertible, then  $f$  is invertible.

9. How many elements are there in  $\text{SL}(3, 7)$ ?