Problem 1. The *Hessian* matrix of a function $f : \mathbb{R}^2 \to \mathbb{R}$ at a point $p \in \mathbb{R}^2$ is defined to be

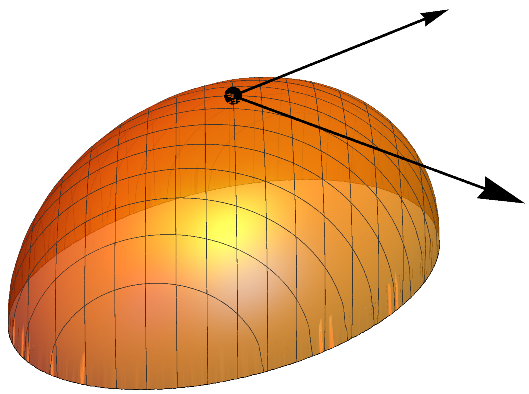
$$H_p = \begin{pmatrix} \frac{\partial^2 f(p)}{\partial x^2} & \frac{\partial^2 f(p)}{\partial y \partial x} \\ \frac{\partial^2 f(p)}{\partial x \partial y} & \frac{\partial^2 f(p)}{\partial y^2} \end{pmatrix}$$

You can think of the Hessian as a kind of second derivative of f since it's the derivative of the function $\mathbb{R}^2 \to \mathbb{R}^2$ defined by $p \mapsto \left(\frac{\partial f(p)}{\partial x}, \frac{\partial f(p)}{\partial y} \right)$.

Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x, y) = 6\sqrt{36 - 9(x - y + 1)^2 - 4(x + y - 3)^2}.$$

There is one critical point $p \in \mathbb{R}^2$ of f. Find the eigenvectors of the Hessian at that critical point and explain why they are orthogonal.



Problem 2. Find a 3×3 matrix A with nonzero integer entries so that $A^2 = I$

Problem 3. Describe all the different similarity classes of matrices that have the characteristic polynomial $\chi(t) = t^5 - 2t^3 + t$? Which are invertible?

Problem 4. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (u, v)$ where

$$u = \sin(x + y)$$
$$v = \exp(x - 2y)$$

Notice that $(0, 0) \overset{f}{\mapsto} (0, 1)$. In a neighborhood of the point $(x, y) = (0, 0)$, the function f is invertible. That is, there exist an open set U containing $(u, v) = (0, 1)$ and a function $g : U \to V$ so that $g(u, v) = (x, y)$.

Use the fact that f and g are inverses to compute the total derivative Dg implicitly at the point $(u, v) = (0, 1)$.

Problem 5. Compute a determinant. Your choice:

$$\begin{pmatrix} \cos(1) & \cos(6) & \cos(11) & \cos(16) & \cos(21) \\ \cos(2) & \cos(7) & \cos(12) & \cos(17) & \cos(22) \\ \cos(3) & \cos(8) & \cos(13) & \cos(18) & \cos(23) \\ \cos(4) & \cos(9) & \cos(14) & \cos(19) & \cos(24) \\ \cos(5) & \cos(10) & \cos(15) & \cos(20) & \cos(25) \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{9} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \end{pmatrix}$$

EXAM

Final Exam

Math 207: Fall 2014

December 23, 2014

- Do problem 1. Do two more (for an A) or one more (for a B).

Success!