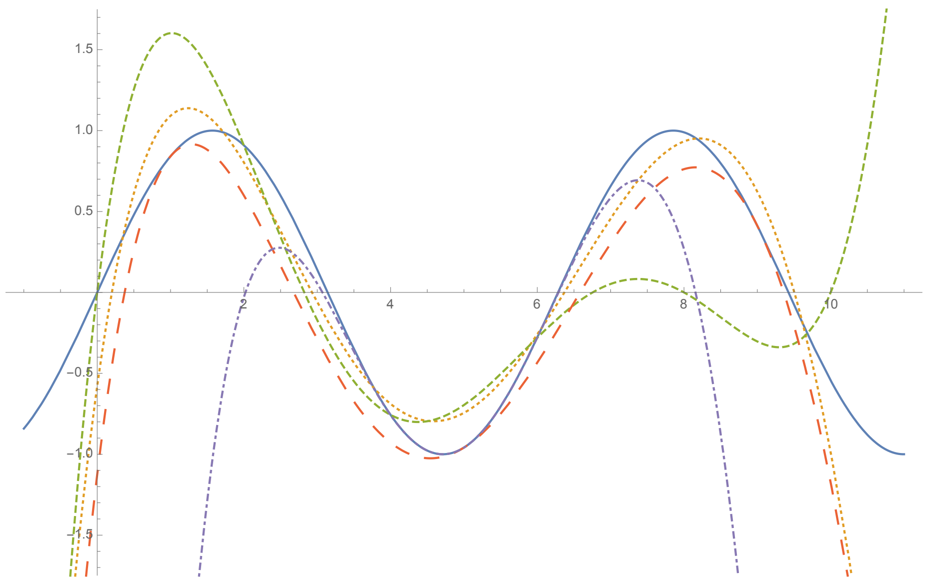
Problem 1. Find a 3×3 matrix A with nonzero integer entries so that $A^3 = A$.

Answer:

Problem 2. Let V be the vector space of five-times differentiable real-valued functions and let W be the subspace of polynomials of degree less than or equal to five. The following formulae define symmetric bilinear functions on V, which restrict to inner products on W.

$$\langle f,g\rangle_1 = \int_0^{10} f(t)g(t)dt$$
$$\langle f,g\rangle_2 = f(0)g(0)+f(2)g(2)+f(4)g(4)+f(6)g(6)+f(8)g(8)+f(10)g(10)$$
$$\langle f,g\rangle_3 = f(1)g(1)+f(5)g(5)+f(9)g(9)+f'(1)g'(1)+f'(5)g'(5)+f'(9)g'(9)$$
$$\langle f,g\rangle_4 = f(5)g(5)+f''(5)g''(5)+f''(5)g''(5)+f'''(5)g'''(5)+f''''(5)g''''(5)+f'''''(5)g'''''(5)$$

The following picture shows the function $f(t)=\sin(t)$ together with several degree five polynomials obtained by projecting f onto W using the given inner products. Which curves go with which inner products? Justify your answer, but don't do any computations!



Answer:

Problem 3. Let

$$M = \begin{pmatrix} 4 & -5 & -2 & 9 \\ -5 & 7 & 5 & -10 \\ 3 & -5 & -3 & 7 \\ -4 & 5 & 3 & -8 \end{pmatrix}.$$

Prove that M is not diagonalizable over the real numbers but M is diagonalizable over the complex numbers.
Hint: Compute M^2.

Answer:

Problem 4. Compute e^B where $B = \begin{pmatrix} -2\pi & -\pi \\ 5\pi & 2\pi \end{pmatrix}$.

Answer:

Problem 5. Let V be the vector space of polynomials of degree less than or equal to two and define an inner product on V by

$$\langle p, q\rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

A convenient orthonormal basis for V is

$$B = \left\{ \frac{1}{2}(t-1)t, 1-t^2, \frac{1}{2}t(t+1) \right\}.$$

Consider $T : V \to V$ defined by $T(p) = p'(t)$. Let T^* denote the adjoint of T.

Find $T^*(t)$.

Answer:

EXAM

Linear Algebra Exam

Math 207: Fall 2014

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- There are five problems. Do three for an A, do two for a B.

Success!