Problem 1. Let $S = \{\heartsuit, \diamondsuit, \clubsuit, \spadesuit\}$ and define two binary operations $+$ and $\times$ as follows:

$+$	$\heartsuit$	$\diamondsuit$	$\clubsuit$	$\spadesuit$
$\heartsuit$	$\heartsuit$	$\diamondsuit$	$\clubsuit$	$\spadesuit$
$\diamondsuit$	$\diamondsuit$	$\heartsuit$	$\spadesuit$	$\clubsuit$
$\clubsuit$	$\clubsuit$	$\spadesuit$	$\heartsuit$	$\diamondsuit$
$\spadesuit$	$\spadesuit$	$\clubsuit$	$\diamondsuit$	$\heartsuit$

$\times$	$\heartsuit$	$\diamondsuit$	$\clubsuit$	$\spadesuit$
$\heartsuit$	$\heartsuit$	$\heartsuit$	$\heartsuit$	$\heartsuit$
$\diamondsuit$	$\heartsuit$	$\diamondsuit$	$\heartsuit$	$\diamondsuit$
$\clubsuit$	$\heartsuit$	$\heartsuit$	$\clubsuit$	$\clubsuit$
$\spadesuit$	$\heartsuit$	$\diamondsuit$	$\clubsuit$	$\spadesuit$

(a) Which element of S is an identity for the operation $+$?

(b) Solve the equation $\diamondsuit x + \clubsuit = \spadesuit$ for x.

(c) Only one of the field axioms is not satisfied by S with $+$ and $\times$. Which one?

Problem 2. Let A, B and C be sets.

(a) One of the following conditions is sufficient for $(A \setminus B) \setminus C = A \setminus (B \setminus C)$. Which one?

- $A \subset (B \cup C)$
- $(B \cup C) \subset A$
- $A \cap B \cap C = \emptyset$
- $C \subset (B \setminus A)$
- $A \cap B = C \cap B$

(b) Prove that the condition you identified implies that $(A \setminus B) \setminus C = A \setminus (B \setminus C)$.

(c) Give an example to show that the identified condition is not *necessary* for $A \setminus B = A \setminus C$.

Problem 3. True or False. Give brief, but conclusive evidence, to support your answer.

(a) For all sets S and for all $A \subseteq S$ there exists a unique set $B \subseteq S$ with $A \cup B = S$.

(b) For all sets $A \subseteq \mathbb{R}$, either A or $\mathbb{R} \setminus A$ is bounded above.

(c) For all $x, y \in \mathbb{R}$, if $x^2 < y^2$ then either $x < y$ or $-x < y$.

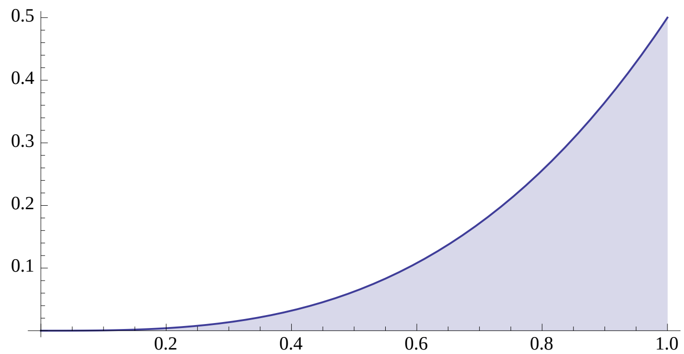
(d) For all $x \in \mathbb{R}$ and for all $n \in \mathbb{N}$ $\left(|2x - 6| < \dfrac{1}{n} \Rightarrow x = 3 \right)$.

Problem 4.

(a) Use mathematical induction to prove that $\sum_{k=1}^{n} k^3 = \frac{(n)^2(n+1)^2}{4}$ for all $n \in \mathbb{N}$.

Problem 4.

(b) Use this result to compute the area of the region pictured below (the vertical distance between the point b units from 0 is $\frac{1}{2}b^3$).



EXAM

Exam 1

Math 157

Thursday, October 3, 2013

- This exam is due at the beginning of class on Tuesday, October 8.
- You are allowed to use your book or your notes, but you may not consult with any person about the exam, or use the internet as a resource for the exam.
- Each part of each problem is worth one point for a total of twelve possible points.

Success!