Problem 1. Let $x \geq -1$. Prove that for all $n \in \mathbb{N}$ we have $(1+x)^n \geq 1 + xn$.

Answer:

Problem 2. Let X and Y be sets and $f : X \to Y$ be a function. Consider the following argument:

> *Suppose that $f : X \to Y$ is not injective. Then there exist elements $x, z \in X$ with $f(x) = f(z)$ and $x \neq z$. Let $A = \{x\}$. Note that $z \in X \setminus A$ so*
>
> $$f(z) \in f(X \setminus A).$$
>
> *Since $x \in A$, $f(x) \in f(A)$, so*
>
> $$f(x) \notin Y \setminus f(A).$$
>
> *Since $f(x) = f(z)$, we've found an element in $f(X \setminus A)$ that is not in $Y \setminus f(A)$. Therefore, $f(X \setminus A) \nsubseteq Y \setminus f(A)$.*

Which of the following propositions does the argument above prove?

(a) If f is injective, then for all $A \subseteq X$ we have $f(X \setminus A) \subseteq Y \setminus f(A)$.

(b) If f is injective, then there exists a set $A \subseteq X$ with $f(X \setminus A) \subseteq Y \setminus f(A)$.

(c) If for all subsets $A \subseteq X$ we have $f(X \setminus A) \subseteq Y \setminus f(A)$, then f is injective.

(d) If f is injective, then for all $A \subseteq X$ we have $Y \setminus f(A) \subseteq f(X \setminus A)$.

(e) If for all sets $A \subseteq X$ we have $Y \setminus f(A) \subseteq f(X \setminus A)$, then f is injective.

(f) If f is not injective, then for all $A \subseteq X$ we have $f(X \setminus A) \nsubseteq Y \setminus f(A)$.

Problem 3. Give an example of a function $f : X \to Y$ and a set $A \subseteq X$ for which

$$f^{-1}(f(A)) \neq A.$$

Answer:

Problem 4. True or False. Right answer $+1$, wrong answer -1, no answer 0.

(a) The field axioms of $\mathbb{R}$ imply that $1 + 1 \neq 0$.

(b) For all functions $f : X \to Y$ and for all sets $C \subseteq Y$, we have $f^{-1}(Y \setminus C) = X \setminus f^{-1}(C)$.

(c) For all functions $f : X \to Y$ and for all sets $C \subseteq Y$, we have $f(f^{-1}(C)) = C$.

(d) For all injective functions $f : X \to Y$ and for all sets $A \subseteq X$ we have $f^{-1}(f(A)) = A$.

EXAM

Exam 2

Math 157

Thursday, October 17, 2013

- You have one hour to complete this exam.
- Problems 1 and 4 are worth 4 points, Problem 2 and Problem 3 are worth 2 points for a total of 12 possible points. For a bonus point, define ”lapidate.”

Success!