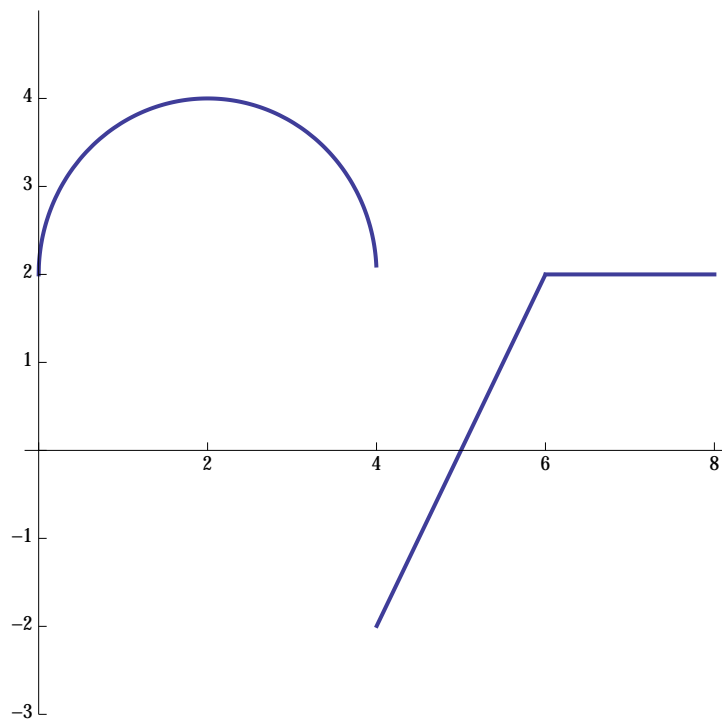

Definitions and theorems [2 points each]

Problem 1. Let f be a bounded function defined on $[a, b]$. Define the statement “ f is integrable” and the number $\int_a^b f$.

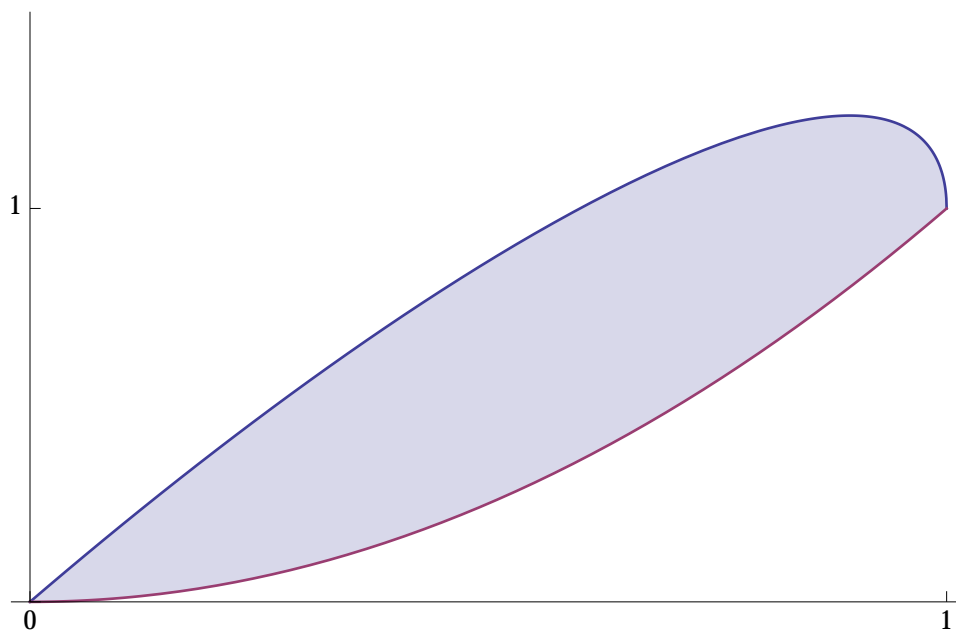
Problem 2. Let f be a function defined on an open neighborhood of c . Define the statement “ f is continuous at c .”

Multiple Choice [1 point each]**Problem 3.** Below the graph of a function f is sketched

$$\int_2^8 f(t) dt =$$

- (a) $\pi + 6$
 - (b) $\pi + 8$
 - (c) $\pi + 10$
 - (d) $\pi + 12$
 - (e) $\pi + 14$
-

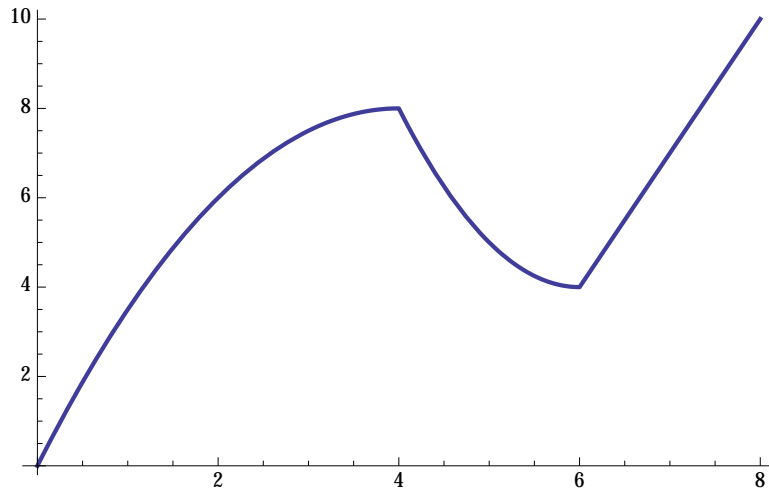
Problem 4. Consider the region sketched below.



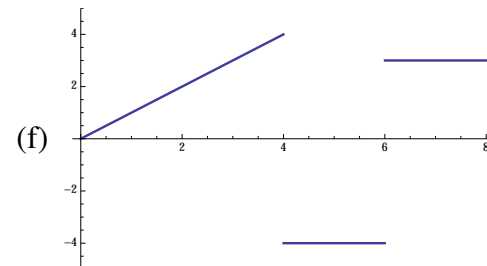
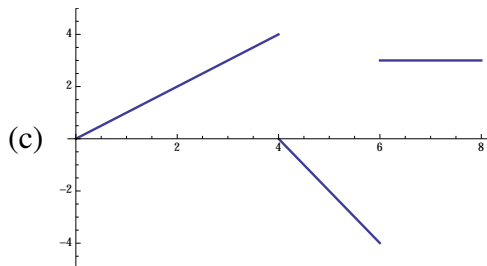
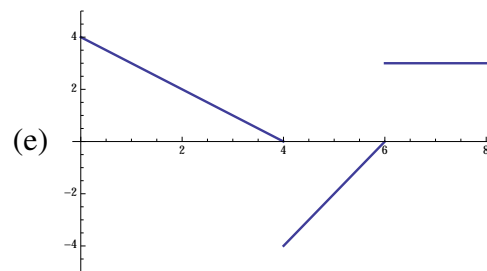
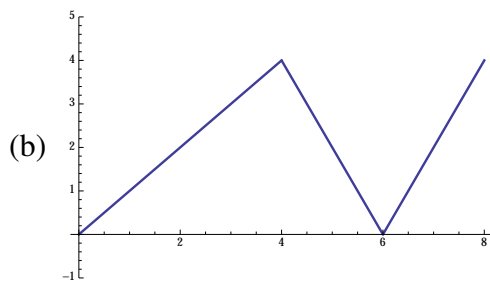
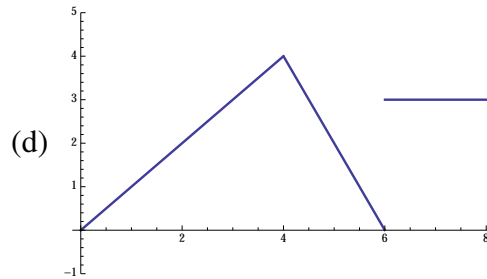
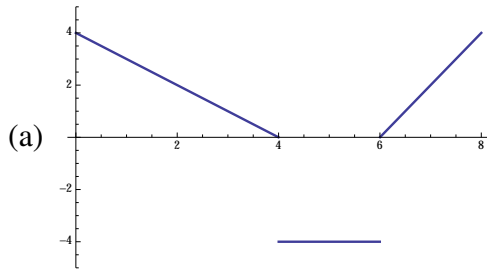
The curve on top is defined by $y = 2x + \sqrt{1 - x^2} - 1$ and the curve on bottom is $y = x^2$. The area of this region is

- (a) $\frac{\pi}{4} - \frac{2}{3}$
 - (b) $\frac{\pi}{4} - \frac{1}{2}$
 - (c) $\frac{\pi}{4} - \frac{1}{3}$
 - (d) $\frac{\pi}{4} + \frac{1}{4}$
 - (e) $\frac{\pi}{4} + \frac{1}{2}$
-

Problem 5. Here's the graph of $A(x) = \int_0^x f(t)dt$:



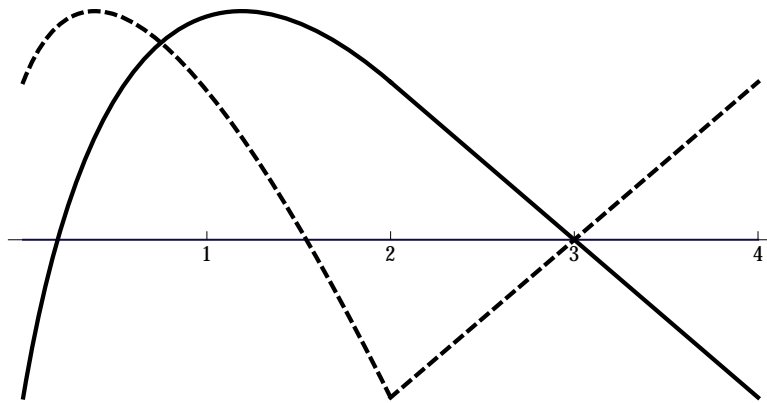
Which is the graph of f ?



Problem 6. On the interval $[0, 4\pi^2]$ the function defined by $A(x) = \int_0^x \sin(\sqrt{t}) dt$ has a global maximum at

- (a) $\frac{1}{4}\pi^2$
- (b) π^2
- (c) $4\pi^2$
- (d) $\frac{25}{4}\pi^2$
- (e) $\sqrt{2\pi}$

Problem 7. The graphs of two functions are sketched below.



The graph of f is solid and the graph of g is dashed. $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)} =$

- (a) 0
- (b) 1
- (c) -1
- (d) 2
- (e) does not exist

Problem 8. Which statement is false?

- (a) There exists a continuous injection $f : (0, 1) \rightarrow [0, 1]$.
 - (b) There exists a continuous surjection $f : (0, 1) \rightarrow [0, 1]$.
 - (c) There exists a continuous surjection $f : [0, 1] \rightarrow (0, 1)$.
 - (d) There exists a continuous injection $f : [0, 1] \rightarrow (0, 1)$.
-

Problem 9. Which statements about a function $f : X \rightarrow Y$ could be false?

- (a) If f is injective, then for all sets $A, B \subseteq X$ we have $f(A \cap B) \subseteq f(A) \cap f(B)$.
 - (b) If f is injective, then for all sets $A, B \subseteq X$ we have $f(A) \cap f(B) \subseteq f(A \cap B)$.
 - (c) If $f(A \cap B) = f(A) \cap f(B)$ for all sets $A, B \subseteq X$ then f is injective.
 - (d) If f is not injective then there exist sets $A, B \subseteq X$ with $f(A \cap B) \neq f(A) \cap f(B)$
 - (e) If f is surjective then for all sets A we have $f(X \setminus A) \subseteq Y \setminus f(A)$.
-

Problem 10. Let

$$g(x) = \begin{cases} x & \text{if } x \text{ is irrational} \\ -x & \text{if } x \text{ is rational.} \end{cases}$$

Which of the following statements is false?

- (a) g is continuous at 0
 - (b) $\lim_{x \rightarrow 0} g(x) = 0$
 - (c) g is invertible
 - (d) g is bounded on $[-1, 1]$
 - (e) g is piecewise monotonic
-

Problem 11. Let

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \text{ in lowest terms.} \end{cases}$$

Which of the following statements is false?

- (a) $f(0) = 1$
 - (b) $\lim_{x \rightarrow p} f(x) = 0$ for every number p
 - (c) f is continuous at every irrational number
 - (d) f is invertible
 - (e) f is integrable on $[0, 1]$
-

Problem 12. Suppose that $f : [0, 1] \rightarrow [0, 1]$ is a continuous function satisfying $f(0) = \frac{1}{2}$ and $f(1) = \frac{1}{2}$. Which of the following statements *must be* false?

- (a) $0 < \int_0^1 f < 1$
 - (b) f is invertible
 - (c) there is a number $c \in [0, 1]$ with $f(c) = 0$
 - (d) f is bounded
 - (e) there is a number c with $f(c) = 0$
-

Problem 13. Suppose that $f : [0, 1] \rightarrow [0, 1]$ is a continuous function satisfying $f(0) = \frac{1}{2}$ and $f(1) = \frac{1}{2}$. Which of the following statements *might be* false?

- (a) $0 < \int_0^1 f < 1$
 - (b) f is invertible
 - (c) there is a number $c \in [0, 1]$ with $f(c) = 0$
 - (d) f is bounded
 - (e) there is a number c with $f(c) = 0$
-

Matching computations [1 point each]

14. $\lim_{h \rightarrow 0} \frac{1}{h} \left(\cos \left(\frac{\pi}{6} + h \right) - \frac{\sqrt{3}}{2} \right)$

15. $\int_0^\pi \left| \cos(t) + \frac{1}{2} \right| dt$

16. $\int_0^6 [\sqrt{x}] dx$

17. $\sin \left(\frac{\pi}{12} \right)$

18. $\int_0^{\pi^2} \sqrt{x} dx$

The answers (out of order) are:

(a) $\frac{2\pi^3}{3}$

(b) 7

(c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(d) $\sqrt{3} + \frac{\pi}{6}$

(e) $-\frac{1}{2}$

Bonus [1 point]

Let $f(x) = x^3$. Give a rigorous, epsilon-delta proof that the function f is continuous at 1.

Answer:

EXAM

Final Exam

Math 157

Tuesday, December 17, 2013

- Make sure your solutions are clearly and carefully written.
Proofread.
- There are 20 points and 1 bonus points (a total of 21 possible).

May the Force be with you!