Math 157 Fall 2013 — Homework 3

Problem 1. Let A, B and X be sets. Prove that

$$X - (A \cup B) = (X - A) \cap (X - B).$$

Problem 2. Construct a truth table for the following compound propositions.

(a) $(p \wedge q) \Rightarrow r$

(b) $(p \vee q) \Rightarrow r$

(c) $(p \Rightarrow q) \Rightarrow r$

(d) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \wedge \neg q) \Rightarrow r)$

Problem 3. We say two propositions p and q are logically equivalent if they have the same truth values and write $p \equiv q$. This is the same as saying that the biconditional $p \Leftrightarrow q$ is a tautology, i.e., always true. Prove or disprove each of the following logical equivalences. You may use truth tables, or you may use various laws of Boolean algebra (see for example, the monotone and nonmonotone laws at `http://en.wikipedia.org/wiki/Boolean_algebra_(logic)` .)

(a) $p \Rightarrow q \equiv \neg p \vee q$

(b) $(p \Rightarrow q) \vee (p \Rightarrow r) \equiv p \Rightarrow (q \vee r)$

(c) $(p \Rightarrow r) \vee (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$

(d) $(p \Rightarrow q) \Rightarrow (r \Rightarrow s) \equiv (p \Rightarrow r) \Rightarrow (q \Rightarrow s)$

Problem 4. Find a compound proposition involving propositions p, q, and r that is true when exactly one of them is true and false otherwise.

Problem 5. Let $\mathbb{R}$ be the set of real numbers, $[0,1] = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$, and $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ be the set of natural numbers. Negate the following propositions:

(a) $\forall x \in \mathbb{R} \exists y \in \mathbb{N} (y > x)$

(b) $\exists y \in \mathbb{R} \forall x \in \mathbb{N} (y > x)$

(c) $\forall \epsilon > 0 \exists N \in \mathbb{N} \forall n \in \mathbb{N} (n > N \Rightarrow \frac{1}{n} < \epsilon)$

(d) $\forall x \in [0,1] \forall \epsilon > 0 \exists N \in \mathbb{N} \forall n \in \mathbb{N} (n > N \Rightarrow x^n < \epsilon)$

Problem 6. Decide whether each of the propositions in the previous problem is true or false.

Problem 7. Compare the following algebraic structures:

- *Sets* together with the binary operations $\cup$ and $\cap$.
- *The real numbers* $\mathbb{R}$ together with the binary operations $+$ and $\times$.
- *Propositions* together with the binary operations $\vee$ and $\wedge$.