Math 157 Fall 2013 **Homework 5**

Problem 1. Read the rest of chapter I (through page 43).

Problem 2. Find the least upper bound of the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$. Prove your answer.

Problem 3. True or False:

(a) $p \Rightarrow (q \vee r) \equiv (p \wedge \neg q) \Rightarrow r$

(b) $(p \wedge q) \Rightarrow r \equiv (p \wedge \neg r) \Rightarrow \neg q$.

Problem 4. Prove that

If $a, b, c > 0$ and $a + b + c = 1$ then $(1 - a)(1 - b)(1 - c) \geq 8abc$.

Problem 5. $n!$ may be defined inductively for $n = 0, 1, 2, \ldots$ by

- $0! = 1$ and
- $n! = n \times (n - 1)!$

For $n, k = 0, 1, 2, \ldots$, define $\binom{n}{k}$ by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Prove that

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

Problem 6. Prove that for all $x \in \mathbb{R}$, $0 \leq |x| - x \leq 2|x|$.

Problem 7. Prove that for all $n \in \mathbb{N}$, $\displaystyle\sum_{k=1}^{n} 2k - 1 = n^2$.