Math 157 Fall 2013 **Homework 7**

Step functions

Problem 1. Read sections 1.8-1.14 (pages 60-70) in Apostol and do the following exercises:

(a) Exercises 1, 3 in 1.11 on page 63.

(b) Exercises 1, 2, 5, 11, 13-17 in section 1.15 on pages 70-72.

More on abstract functions

Definition 1. Suppose that $f : X \to Y$ and $g : Y \to Z$ are functions. We define the composition $g \circ f$ to be the function

$$g \circ f : X \to Z$$

given by $(g \circ f)(x) = g(f(x))$.

Definition 2. For any set X, define the function $\mathrm{id}_X : X \to X$ by $\mathrm{id}_X(x) = x$ for all $x \in X$.

Problem 2. Prove that $\circ$ is associative. That is, if $f : X \to Y$, $g : Y \to Z$, and $h : Z \to W$ are functions, then

$$(h \circ g) \circ f = h \circ (g \circ f).$$

Problem 3. Prove that for any function $f : X \to Y$, we have

$$f \circ \mathrm{id}_X = f \text{ and } \mathrm{id}_Y \circ f = f.$$

Definition 3. Let $f : X \to Y$ be a function. We say that a function $g : Y \to X$ is a *left inverse* of f if $g \circ f = \mathrm{id}_X$. We say that a function $g : Y \to X$ is a *right inverse* of f if $f \circ g = \mathrm{id}_Y$. We say that a function $g : Y \to X$ is *an inverse* of f if g is both a left and a right inverse of f.

Problem 4. Prove that $f : X \to Y$ has a left inverse if and only if f is injective.

Problem 5. Prove that $f : X \to Y$ has a right inverse if and only if f is surjective.

Problem 6. Prove that if $f : X \to Y$ has a left inverse $g : Y \to X$ and a right inverse $h : Y \to X$, then $g = h$.